DUVAL R. Cognitive conditions of the geometric learning : developing visualisation, distinguishing various kinds of reasoning and co-ordinating their running p. 5 - 53

Geometry is a kind of knowledge area that requires the cognitive joining of two representation registers: on one hand the visualisation of shapes in order to represent the space and on the other hand the language for stating some properties and for deducing from them many others. The troubles of learning first come from the fact these two registers are used in a way which is opposite to their cognitive use apart from mathematics.

The way of seeing a geometrical figure depends on the activity for what it is used. Thus it can run in an iconic way or in a non-iconic way. The non-iconic visualisation involves that the first recognised shapes would be visually deconstructed. There are three kinds of shape deconstruction: deconstruction by using tools in order to construct any figure, the heuristic breaking down in order to solve problems and the dimensional deconstruction. This one is the central process of geometrical visualisation.

The analysis of language in geometry requires that three levels of discursive operations would be separated: verbal designation, property statement and deduction. This separation is important because the relation between visualisation and language changes completely from one level to the other. This variation conceals the most important cognitive phenomenon: the dimensional hiatus. Comings and goings between visualisation and language involve a jump into the number of dimensions in order to recognise the knowledge objects that are represented within each register.

Becoming aware of the dimensional deconstruction of shapes and understanding the process of the various discursive operations are the conditions for succeeding in making the two registers run in synergy. There are the crucial thresholds for learning in geometry. Is that really taken into account in the teaching?

Key words. Functional analysis, code, visualisation, counter-example, heuristic figural decomposition, dimensional building, definition, straight line, figure, proof, proposition, figural unit.

TANGUAY D. Learning formal proof by organizing oriented graphs: an experiment p. 55 - 93

According to Duval, traditional tasks of analysing, reading or writing proofs don't enable pupils to discriminate between formal proof a (logical) *computation,* where each proposition obeys criteria of *validity* and is brought in for its *operative status* and argumentation a *discourse*, where propositions obey criteria of *relevance* and are organised by simple *accumulation*.

Pushing more radically Duval's proposed research orientations and trying as much as possible to *isolate difficulties*, we have designed tasks in which pupils organise, within the empty boxes of an oriented graph, the propositions of a geometrical proof, the general idea of which they have been previously introduced to. The sequence of tasks has been experimented with three 1^{st} year secondary classes (12-13 years old) in Montreal, in the spring of 2004.

A first analysis of the data that have been gathered allows us, among other things, to conclude that:

- the deductive structure, consisting of chained inferences, is neither spontaneously, nor easily understood by pupils;
- the passage from what seems to be a satisfactory understanding of a proof, of how the general ideas are linked in it, to a logically well-controlled written production of that proof, constitutes a fundamental leap for pupils and deeply hinges on their mastery of the deductive structure;
- the organizational work required by the proposed tasks may contribute to pupils' better understanding of the mechanisms that rule this structure.

Key words. Secondary school, geometry, proof, deductive step, inference, logical linking, proof graph, writing

ADJIAGE R. Diversity of the ratio problems p. 95 - 129

Concerning the problem solving by 9-to-13 year-old pupils, in the case of quantity ratio and proportionality, researchers have considered numerous variables. Strangely, one of them, the physical-empirical context often referred to, is little taken into account. The goals of this paper are to better define the values of this variable and validate its pertinence. I thus first made a classification of the involved problems, providing the values of the new variable. A questionnaire was then elaborated. It presents for solution six items, one representative of each of the six problem-types of the classification. The survey was conducted with two types of population: 12-to-13 year-old pupils, and primary school teacher-trainees as a reference population. The findings show that the variations (success rate and procedures used) are important from one item to the other in the pupil group, and slight in the teacher-trainee group. These variations are attributed to the physical context, as the other variables are constant in the questionnaire. This study, along with other previous researches, allows us to draw the complexity of ratio problems closer around two principles of separation / articulation between the physical and the mathematical worlds, and between the rational registers. Furthermore, it permits to consider that the diversity of those problems lies in the physical-empirical domain, while their unity can be located in the mathematical one.

Key words. Fraction, proportionality, physical context, quantity, ratio, semiotic register.

WINSLØW C. Defining goals of mathematics education: the contents - competencies dialectic p. 131 - 155

The description of goals of mathematics education has several potential ends, external (justification, declaration) as well as internal (planning, evaluation). Although these ends are not independent, the usual forms of description have a tendency to serve but a part of these ends. In this paper, we examine the notion of "competency" as a possible solution to these problems. We also consider some examples of its use especially in the Danish context.

Key words. Mathematics education, curriculum, contents, competencies

TRIGUEROS M. & OKTAÇ A. APOS Theory and the Teaching of Linear Algebra p. 157 - 176

In this article we present the APOS (Action - Process - Object - Schema) theoretical framework and we explain its use in the case of a research project involving mental constructions that linear algebra students make while learning this subject. To illustrate the use of the theory we have chosen the concept of vector spaces, as it is one of the fundamental concepts of linear algebra and since in the approach that we describe, it constitutes the beginning of an introductory course.

Keywords. APOS theory, linear algebra, vector spaces.

CUEVAS V., A., MORENO G., S. & PLUVINAGE F. Experimenting an Object-Oriented Teaching of Function p. 177 − 207

Previous investigation has reported that students, instructed about the theorem of maximum and minimum reached by a continuous function on a closed interval, most generally adopt an algorithmic conduct applying it to a differentiable function: They start from an algebraic expression and obtain its critical values, even if those lie outside of the interval to be considered. Our hypothesis was this conduct, alike to different manifestations mentioned by other searchers, should result from a lack of meaning in the concept of function itself. Therefore, we have designed an experiment, intending to expand the meaning of the concept of function. We chose, as an *action project*, the modelling of an easy-to-understand physical situation, an inflatable sphere in a recipient containing some liquid, which allowed introducing a function in a not merely algebraic form, but in a way more similar to the historical approach that implies to study interconnected variables. We decided to implement three successive stages in our experiment: descriptive, quantitative, and generalizer. The purpose was to favour the use of various semiotic registers in order to express functions and their properties. After the two-week experiment, the assessment clearly demonstrates students' enhancement in the knowledge of functions. Most (17 out of 21) succeeded in partially or totally solving a problem reported in previous studies as mishandled.

Key words. Calculus, function, function concept, extreme values, variables, modelling, action project, semiotic registers, teaching experiment.

ROBERT A. From research works on teachers' practice to training secondary school mathematics teachers: a didactical approach p. 209 - 249

In this text, we first discuss the legitimacy of our project and then we present some research works we have obtained on mathematics teachers' practices. We infer some propositions on teachers' training. They concern trainings modes and methods, but they fit only teachers' training which have some direct relation with classrooms practices. We defend the idea that it is very difficult for a teacher to change his own practices: this comes from our researches results on practices stability, based on complexity and individual coherence. Furthermore, all teachers as "workers" share some institutional and social constraints and the common answers they have develop are also very stable ones. That is why we insist on teachers trainings modes and methods. We present then a precise project called "scenario", based on classroom videotapes analyses. The aim of the training is to give teachers tools to analyse these videos. We try to show how this project fits our proposals: long training, work on both contents and classrooms management, real and tied to professional interest activities with a collective dimension, allowing a real work on constraints and choices. To conclude, we evoke some research we think it would be useful to conceive, namely on evaluation of trainings teachers.

Key words. Secondary mathematics teachers training, teachers practices, mathematics teachers trainers training, "scenario", hypotheses on trainings teachers.