ANNALES de DIDACTIQUE et de SCIENCES COGNITIVES, Volume 11, 2006, IREM de STRASBOURG
ABSTRACTS

Abstract. This paper starts from the assumption that every reform of mathematics teaching is based on an ideology, i.e. on a vision of a desirable state of this teaching contrasted with its undesirable but existing state. The question asked is: What are the obstacles to the realisation of this desirable state and what contributes to the tenacity of the non-desirable state? In the main section of the paper, I take this question to review a few idealistic postulates regarding the goals of mathematics teaching, the coordination of curricula and relevance of the mathematical content, the teaching for understanding and autonomy of thought, the problem solving approach to developing students' mathematical thinking, students' motivation, assessment and use of technology. The last section is devoted to a reflection on the possibilities of communication between ideology and theory, in spite of the important differences between the two. I propose that a didactic theory specific to mathematics could constitute a useful channel for this communication, by turning the ideological debate into a pragmatic debate, based on theoretical arguments and questions such as, what are the possible (rather than "desirable" or "good") choices or solutions of a didactic, pedagogical or institutional problem? What do we know about the possible consequences of each choice, based on the available evidence from experience and research?

Abstract. This rather speculative paper proposes an overarching theoretical perspective for characterizing human decision-making and problem solving in the moment. The scope is deliberately broad. My intention is to address the following question: How and why do people make the decisions they do, as they are engaged in acts of problem solving?

Some fundamental assumptions in this enterprise are:

1. Problem solving is used in a deliberately broad way here. It includes a child's actions in interacting with its parents, a student working on a mathematics problem in class or in the laboratory, and a teacher's decision-making while teaching a mathematics (or other) lesson. More broadly, I assume that almost all human action is goal-oriented and that attaining high-priority goals can be characterized as a problem.

2. Most human behavior is rational, in the following sense. The actions people take in any particular context are fundamentally aimed at solving problems that are important to them. (These may or may not be the problems they have been assigned to solve!) If one is capable of understanding what problem a person is trying to solve at any given time, that person's actions will often be seen to be rational and consistent. In certain contexts, such as teaching and problem solving (by the standard definition), that consistency in behavior can be strong enough to allow the individuals actions to be modeled.

3. In any given context, decision-making is a function of beliefs, goals, and knowledge. In brief outline: an individual's beliefs, in interaction with the context, shape the formation and prioritization of goals. Given a particular constellation of goals, the individual looks for and implements knowledge that is consistent with his or her belief systems and is designed to

satisfy one or more high-priority goals. As goals are satisfied (or not), or as the context changes, new goals take on high priority, and actions are then taken in the pursuit of these goals.

Examples are given to suggest the way in which this theoretical perspective can play out.

JEAN-CLAUDE RAUSCHER ***Reflexive writing and mathematical activity: case of proportion problem solving,*** **p. 75 − 102.**

Abstract. How can the writing of a pupil's reflections contribute to the development of his or her knowledge? This question is considered at the end of primary school in an experiment on problem solving about proportionnality. The problems consist of comparing mixtures, and they are inspired by Noelting's problems (1980) about orange juices. Based on the work of Duval (1998), our observations show a conceptual obstacle when the texts written by the pupils are used in class to pass from a situation of expression to a situation of proof : the pupils consider the written texts as spoken expressions. Spontaneously, the pupils write as they speak, never going back to what they have written. Our experiment aims to lead the pupils to consider their texts really as written productions and not as simple transcriptions of oral expressions. We analyse the extent to which this aim is achieved, the consequences on the nature of the proofs produced by the pupils, and the development of their knowledge. Finally, we indicate other experiments which can be carried out with the pupils according to our observations.

PATRICIA MARCHAND ***Developing mental imagery linked to spatial geometry learning*****, p. 103−121.**

Abstract. This article puts forward specific results from our interdisciplinary Ph. D. thesis, involving mathematics and sports, in order to progress towards a framework for the elaboration of activities aiming to develop three dimensional mental images. We illustrate this framework with examples of activities from elementary and secondary level.

JORGE SOTO-ANDRADE ***A world in a grain of sand: Metaphors and analogies in mathematics Education,*** **p. 123−147.**

Abstract. Metaphors are not just rhetorical devices, but also very powerful cognitive tools. We can check through learning experiments that they help us, not only to learn new ideas, but also that they provide very efficient means to calculate. They have a bigger cognitive thrust than analogies, which are more familiar in didactics: A metaphor is an analogy that doesn't warn you in advance! We present here concrete examples that bring grist to our mill

Keywords: Mathematics education, metaphor, analogy, comparison, representation, cognition.

ERICH CH. WITTMANN **Mathematics as the Science of Patterns,** ***A guideline for Developing Mathematics Education from Early Childhood to Adulthood,*** **p. 149−174.**

Abstract. The aim of the present paper is to give an account of the holistic approach to mathematical education developed in the project *Mathe 2000*. The emphasis is on the mathematical roots. It will be shown how a special view of mathematics as the science of patterns can be made practical in developmental research. The paper consists of three sections: A short introduction into the project's philosophy in the first section is followed by the description of some typical learning environments. In the third section some underlying theoretical principles are explained by referring to the learning environments of section 2.

Keywords. Patterns, learning environments, learning by discovery, representations, operative proofs, teacher education.

Abstract. One of the first aim of geometry teaching is certainly that a student can build his/her own proper and effective geometrical working space. Then s/he can understand and solve geometry problems by using this space. But, the problem's interpretation is depending on different geometrical paradigms which are related to the kind of institutions -schools but also countries- where geometry is taught. From the diversity of paradigms it results a great diversity of working spaces: that explains a big number of didactic misunderstandings. In the paper, we clarify the notions of geometrical paradigm and working space. Then we show the possible interest of using and developing these tools for studies in geometry didactics.

Abstract. This presentation considers the biological and mathematical mechanisms involved in the development from the child to a mathematician and theorizes how individuals grow in different ways over a life-time's experience. The theory is then used to respond to questions on the long-term teaching and learning of mathematics over the whole curriculum from child to adult.

Abstract. In this article I will study the role of "monsters" in the history of modern mathematics, - especially in the context of real functions and of polyhedra. We will see that the interest in monsters is a rather recent attitude and that it is related to the idea that mathematics is freely constructed by the human mind. We will ask the question whether this point of view is well adapted to pupils.

Abstract. This paper concerns the activities of a research group in mathematics education about the concept of limit. The main characteristic of this group is that all stages in schooling (primary − 6 to 10 years; middle school - 11 to 13 years, and high school - 14 to 19 years) are represented, both within the group itself and as regards the pupils involved. This vertical cross-section is a resource which is as rare as it is precious. It allows us, indeed, to propose similar activities at different levels, check the evolution of the ideas, techniques and errors of the schoolchildren over time, and observe the effect of the teaching methods on them. With the support of research in mathematics education, we chose some issues, in particular measurement and approximation, which, in our opinion, can favour the gradual early development of the concept of limit. In this context, we would emphasise the importance of approximation as a resource for the long way to build the concept of the limit.

Abstract. The role of mental representations and imagery has been studied for several years in order to explain the processes of constructing concepts and to elucidate the mathematical abilities of students. Searching for new ways for the construction of mathematical concepts and problem solving strategies, the Working group on Representations and Mathematics Visualization of PME-NA, 1998-2002 (see Hitt, 2002) highlighted the importance of semiotic representations while building mathematical concepts, giving a new dimension to research work in mathematics education. Taking into account previous research done by Duval (1993, 1995, 1999) on constructing mathematical concepts, we focused on students' conceptions and on the role of the functional representations (spontaneous representations) used by the students in order to build a mathematical concept. We found that the representations used by the students when constructing a mathematical concept play a significant role and that they are part of their conception. These functional representations are of a kind usually not found nor used by mathematics teachers.

Abstract. In this text, I first evoke discussions developed in the educational community about the specificity of mathematical learning at university level or about the nature of avanced mathematical thinking. Then, to reflect on what is offered by recent didactic research carried out at university level, I focus on three dimensions which, in my opinion, evidence the potential offered by some evolutions in research approaches and interests for setting up more adequately the questions at stake, and for progressing in the knowledge of learning processes and of their . These are the followings :

- the increasing attention paid to flexibility in the study of learning processes;
- the evolution of research paradigms from constructivist approaches towards anthropological and socio-cultural approaches;
- the development of new domains for research, taking the example of research on the mathematical training of engineers.