Alain KUZNIAK , The Mathematical Work Space and its Geneses, 9-24

Abstract. In the paper, the notion of mathematical work space is based on some characteristics brought out by previous studies on geometrical work. The mathematical work space is structured on two fundamental levels: the epistemological level in relationship to mathematical contents and the cognitive level linked to visualization, construction and proof processes. To articulate both levels and come into mathematical work three main geneses are considered: a semiotic genesis, an instrumental genesis and a discursive genesis conveying reasoning.

Athanasios GAGATSIS, Eleni DELIYIANNI, Iliada ELIA & Areti PANAOURA, Exploring Flexibility: The case of the numerical domain, 25-43

Abstract. Many studies investigate and discuss flexibility in mathematics learning. However researchers use the concept starting from different theoretical backgrounds and examine different dimensions leading to different results. The present paper studies the theoretical definitions stated for flexibility which are based on the mental operations, the strategies and the representations. The notion of experimental-operational flexibility is also pointed out. Research results regarding strategy flexibility and representational flexibility in mathematics learning are presented. Didactical implications are discussed and suggestions for further research are introduced.

Iliada ELIA, The role of number line in the solution of additive problems, 45-66

Abstract. This study investigates the effects of the number line on students' solution of additive change problems. We organized an experimental program focusing on the use of the number line in coordination with other representations in additive problem solving and compared its outcomes with the results of the regular mathematical curriculum. The experimental group consisted of 356 students, while the control group included 776 students in Grades 1, 2 and 3 of primary school. The data were collected using change problems of various structures in two types of representation, verbal description and verbal description with number line. Confirmatory Factor Analysis showed that the type of representation had an effect on the solution of additive problems of a complex structure, but not in the solution of simple problems. The experimental group students of Grades 1 and 2 performed better than the control group students in all types of problems irrespectively of the representation. The use of the number line was a complex but an efficient way of solving a number of additive problems. It supported problem solving for representing either the situation or the solution of the problem.

Catherine HOUDEMENT, Hidden knowledge in usual verbal problem solving at primary school, 67-96

Abstract. This paper contributes to determine student's knowledge involved in arithmetic word problem solving, that are ignored by mathematics education research but discriminate reinvestment problems solving students : among these skills the use of a modeling approach (models are the arithmetic operations), different types of controls and the qualification. This paper shows how rich is a clinical approach after the solving session to discover ignored knowledge and feeds current (study of semiotic tools) and upcoming researches (controls and qualification) in relation to flexibility.

Sylvia COUTAT & Philippe R. RICHARD, Dynamic figures in a mathematical workspace for the learning of geometrical properties, 97-126

Abstract. Our paper aims at showing how dynamic figures are useful in the learning of the use of geometrical properties at a high school level, in continuity with the practices inherited during the primary school education. After considering the general contextual rooting of the problem situations while comparing geometrical reality with educational institution, we focus on the student-milieu system and on the connections between the reasoning and the operational dynamic figure. We then present a research framework in order to analyze a geometrical workspace dedicated towards the learning of the use of properties. The workspace is presented in what it has of generic to enhance such learning and it enables us to conclude by some theoretical remarks on the components from the suitable geometrical working space.

Bernard PARZYSZ, Some Educational Questions about Statistics and Probability, 127-147

Abstract. This article goes over some of the ideas which emerged in France about the teaching of probability and statistics in secondary education during the past ten years, and first of all it takes up and develops the idea of an analogy between modelling in probability and in elementary geometry, considered here from the standpoint of paradigms. Another specificity of the domain is that its teaching makes a great use of various representations: tables, varied kinds of graphs, tree diagrams, box-and-whiskers plots and so on, which are a major element of the workspace but the construction and the meaning of which are sometimes problematic. It is the same for the articulation of one register with another, which is most often considered transparent, but obviously is not without causing difficulties, even among teachers. Taking into account the notion of semantic congruence may allow working this aspect more specifically and bring to the fore the isomorphism subjacent to random experiments which seem *a priori* different, thus opening up the way to the notion of probabilistic model.

Fabrice VANDEBROUCK, Perspectives and working domains for studies of functions, 149-185

Abstract. The aim of this paper is to understand and interpret the difficulties that students are facing when learning calculus during the transition from high school to university. We are specifically interested by the study of functions. We first discuss the general background concerning this transition and the notion of function. Then we define quite separate working domains, specific on the one hand of the high school practices and on the other hand of university practices. Finally, we present a study in which we examine the freshmen's difficulties in calculus.

Inés Mª GÓMEZ- CHACÓN & Alain KUZNIAK, Prospective Teachers' Geometric Work Space within Technological and Professional Knowledge, 187-216

Abstract. This article is focused on the study of the geometric work involved in the initial teacher training in a learning environment based on the use of GeoGebra dynamic software. The intention is to identify how three figural, instrumental and discursive reasoning genesis are articulated in the Geometric Work Space and to study the role which GeoGebra plays in the construction of this geometric space. In addition, we explore the influence of software in the step from Geometry I to Geometry II in the performance of the student as viewed by the teacher.

Blanca SOUTO RUBIO & Inés Mª GÓMEZ- CHACÓN, *Visualization at University Level. The concept of integral,* 217-246

Abstract. In recent years, several studies have highlighted the importance of tackling the students' difficulties in understanding of the concept of the integral. This study carried out with the first year students of the Mathematics Degree at the Universidad Complutense de Madrid, presents a deeper insight into these difficulties through data collected from a non-routine problem questionnaire and semi-structured interviews. Some of these difficulties clearly have their origin in the coordination between the analytic and graphic registers. In the analysis of students' use of the graphic register, the distinction between two different functions of images (iconic and heuristic) is exploited productively. Moreover, a specific teaching of visualization is recommended. As a main contribution in this approach two examples of relevant characteristic of visualization that should be taken into account in this proposal are shown: a high cognitive requirement and the need of a global apprehension.