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HOW CAN STUDY AND RESEARCH PATHS CONTRIBUTE TO THE TEACHING OF MATHEMATICS IN AN INTERDISCIPLINARY SETTING?

Abstract. This study investigates the perspectives of using study and research paths (SRP) as a design tool for bidisciplinary work at upper secondary level. This study is using a special kind of diagrams both as tool for SRP design and as a tool to analyse the actual SRP realised with students. Specifically I present the design and realisation of a SRP combining mathematics and biology. The results point to advantages of the SRP approach in terms of the way bidisciplinary work is organised, but also challenges in relation to the design process. As for the last point, the test of the designs raises the question to what degree of detail is it necessary to know the practice and theory of both disciplines in order to formulate questions that help students to develop the intended praxeologies, and also for the weak students to discover the need of mathematics for solving problems in other disciplines.

Key words. Upper secondary level; bidisciplinary work; Study and Research Paths.

Résumé. Comment les Parcours d'Etude et de Recherche peuvent-ils contribuer à l'enseignement des mathématiques dans un contexte interdisciplinaire ? Cette étude examine les perspectives d'utilisation des Parcours d'Étude et de Recherche (PER) comme outil de conception pour du travail bidisciplinaire au niveau secondaire supérieur. Cette étude utilise un type spécial de schémas comme outil à la fois pour la conception de PER et pour analyser le PER réellement réalisé avec les élèves. Plus précisément, je présente la conception et la réalisation d'un PER combinant mathématiques et biologie. Les résultats montrent les avantages de l'approche PER en termes d'organisation du travail bidisciplinaire, mais signalent aussi les conditions à remplir pour la conception. En ce qui concerne le dernier point, le test des réalisations soulève la question du niveau de détail auquel il est nécessaire de connaître la pratique et la théorie des deux disciplines, afin de formuler des questions qui aident les élèves à développer les praxéologies voulues, et aussi permettent aux élèves faibles de découvrir le besoin de mathématiques pour résoudre des problèmes d'autres disciplines.

INTRODUCTION

This study presents the results of testing the design tool called *Study and Research Paths* (SRP) at upper secondary level. The basic idea of a SRP is to organise students' approach to a field of knowledge through meaningful and challenging questions. I describe this tool in more detail in the theory section. SRP has been tested in both monodisciplinary settings (e.g. see Winsløw, Matheron & Mercier, in press) and in bidisciplinary settings (Barquero, Bosch & Gascón, 2007; Thrane,

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2009). The SRP developed by Barquero, Bosch and Gascón (2007) concerned the growth of a population of geese on an isolated island; it does not require students to develop substantial knowledge on population biology, but the motivating problem is clearly extra-mathematical. Thrane (2009) experimented a series of SRP concerning analysis of curves in different sport activities actually, which involve students knowledge of how to perform different sport activities, and the students were supposed to use the mathematical analysis improving their own performance in these activities. In this sense the latter seems to be integrating the two concrete school disciplines more than the first one. This study explores the use of SRP in a bidisciplinary setting combining mathematics and biology where the two disciplines are given equal importance. I am particularly interested in how the bidisciplinary setting can help developing mathematical knowledge – and more concretely, in how a SRP combining mathematics with a discipline like biology could support the learning of mathematics. This is not a new idea and similar ones are presented in (Davison, Miller & Metheny, 1995) and (Czemiak, Weber, Sandmann & Ahern, 1999). What this study offers is a thorough analysis of the students detailed outcomes in terms of presented praxeologies, which illustrates the disciplines and their possible connections regulated by the potentials and limitations of ATD and study and research paths.

This paper is a natural continuation of previous work (Hansen & Winsløw, 2011 and Hansen, 2009), which presented a method to use SRP for analysing bidisciplinary written assignments combining mathematics and history. The study revealed severe challenges for creating bidisciplinary projects, that are well functioning both from the viewpoint of students and teachers. The reason for the identified shortcomings were not just caused by the manifest distance between mathematics and history as disciplines, but also by the fact that the teachers' formulation of the assignments were often leading to a parallel structure in the students' work where the two disciplines were not interacting at all. This was clear already from an *a priori* analysis of the assignments. How the a priori analysis is carried out will be elaborated in the section on methodology.

Context of the study

The institutional frame for the experiments with SRP presented in this paper, was general high school (upper secondary level) in Denmark. In this context, a certain amount of time and lessons are devoted to bidisciplinary work. There are many formal regulations of the bidisciplinary work, which acted as constraints and conditions for the testing of the SRP. The most important condition for our experiment was that the SRP should combine mathematics and biology and that the students should write a bidisciplinary report at the conclusion of their work. The report described in this experiment should prepare the students for writing an autonomous report combining to disciplines (called the "study line project"), which represents a high stake exam at the end of high school, and so it is heavily regulated. Similar the report and this experiment was highly regulated; I give some details in order to allow the reader to grasp the setting of our design. After handing in the report, the students get feedback on their writing from the teachers and they rewrote their report as a 3 pages synopsis to be defended in an oral exam months later. The students must do all work on the first version of the report along with their mandatory classes; after six weeks they get two days off for the final writing. They are allowed to write the report in groups of two students. Each student must hand in at most 10 pages.

After handing in the reports, students should have some kind of evaluation of their work. The rules require that students get a grade for their reports along with comments. These comments must reflect what is expected of the student in their "study line project". Therefore a sheet of comments was created for each student. The comments were formulated with explicit reference to the ministerial guidelines for grading study line projects. This means that the students would get comments from both teachers on the following sentences:

> "To what extent are the questions answered? To what extent does the report fulfil the ministerial aims of the biology teaching? To what extent does the report fulfil the ministerial aims of the mathematics teaching? To what extent are the sections of the assignment mutually coherent? Is the use of notes and citations in the text appropriate? Is the list of references satisfying? What is the overall impression of the assignment?"

Based on the comments, they rewrote their report to the synopsis – a paper containing introduction, research questions, answers to these, conclusion and a section putting the problematique in a broader perspective – used at the oral exam.

On the side of the teachers, none of them have an academic background in both mathematics and biology. The biology teacher is an experienced teacher of biology and geography. He is involved with didactic developments in Danish high school, but not a researcher and without any experience teaching SRP. The mathematics teacher has some years experience in teaching mathematics and physics in Danish High school. She is also a researcher in the field of didactics of mathematics and the author of this paper. Both teachers are the everyday teachers of the class in biology and mathematics respectively. The research part was only conducted by the mathematics teacher, which is reflected in the analysis. The choice of disciplines depends on the disciplines the class specialises in. Therefore it is not likely both disciplines are in the academic background of one teacher. The experiment included the entire class of 25 students.

Theory

The theoretical framework of this study is the anthropological theory (ATD). The key notion is *study and research path*, which is used as a design tool as well as for analysing the outcomes of the student reports, and which we now proceed to explain in more detail.

The notion of SRP was presented by Chevallard around 2004[1] and he describes it as based on what he calls a *generating question*, which will be denoted Q_0. This question must be so strong[2] that students can derive new questions $Q_i´$ from it – here, each index *i* represent a branch of inquiry. The answers to the derived questions add up to an answer for the original question Q_0 (Chevallard, 2006, p. 28). Another requirement for the generating question Q_0 formulated by Barquero, Bosch & Gascón, is that it must be "of real interest to the students ("alive") […]" (2007, p. 3). The research and study process leads to tree diagram of pairs (Q_i, A_i) of question and answers (Barquero, Bosch et Gascón, 2007 and Hansen & Winsløw, 2010), such as the example shown in figure 3 – for simplicity the answers to each question (arising from praxeologies developed by the students) are left out of the diagram.

The notion of inquiry can be interpreted as in inquiry-based mathematics education (IBME), which has been conceptualized by Artique and Blomhøj (2013). As they argue "ATD is also a theoretical frame whose design perspective seems especially adapted to IBME" (Artigue & Blomhøj, 2013, p. 806), and further discusses the potentials and limitations regarding the inquiry reflecting the choice of study and research activity or programme as they call it. The strong link between study and research paths and inquiry-based learning is addressed in (Winsløw, Matheron & Mercier, 2013), although they stress the importance of the study process, which cannot be discarded from the inquiry process.

We now return to our context to explain how SRP fit with the conditions for the bidisciplinary work leading to a synopsis for the oral exam. The students are supposed to get training in applying existing knowledge. In terms of ATD this means activating existing *praxeologies*, a term which indicates a complex system of practical and theoretical knowledge (Chevallard, 1999). The students knew a little on first order differential equations and human physiology, including the nervous system. They are supposed to apply their knowledge in new contexts and

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1 However there has been made different suggestions for the translation of *parcours d'étude et de recherché.* In this paper I have chose to use *study and research paths.*

2 A strong question means that students are able to understand it but unable to deliver a complete answer before studying works of others and use these answers in the formulation of an answer to the generating question.

hopefully get a wider picture of both fields. In terms of ATD this is to develop new (mathematical, biological or bidisciplinary) praxeologies from the existing ones (Barquero, Bosch & Gascón, 2007, pp. 9; Hansen, 2009, p. 53). Another requirement for the assignment is that the students should gain experience with searching for information and resources for answering the assignment questions – and also, where possible, develop answers on their own. This is consistent with what Chevallard calls the dialectic of media and milieu (2006, p. 9) where the student on the one hand is studying existing "works", and at the same time is exploring a problem (in this case, mathematical modelling of the distribution of a drug). It is important to point out the necessity and delicacy of this dialectics (Winsløw, 2011, p. 129): a SRP must include *both* study (of works) and *research* (on problems). The students are supposed to do this since the answers were not directly available in the textbooks. On the contrary the students must study the works of others (the textbooks, new materials from library, internet and likewise), and they have to deconstruct this knowledge, combine this with existing praxeologies in order to develop new praxeologies as answers to questions – formulated by themselves or the assignment questions.

The teaching design

The starting point for testing SRP in the bidisciplinary setting was to formulate a generating question fulfilling the conditions set by the school regulations.

The design was created on the basis of a teaching material for mathematics at upper secondary level, published by Technical University of Denmark. The material deals with the function of painkillers in the body and its' modelling by differential equations (Jónsdottir et al., 2009). The reason for choosing this material as inspiration for the generating question is that many of the students involved in the experiment were interested in biology and wanted to work in the health care system later on. Hence the teachers assumed that these students would find a problem on the dosing of medicine relevant and interesting. This might not give students a better mastery of their immediately lived world but it could help them relate their school knowledge to real uses which, in the end, could fulfil the higher goal of a better mastery of their lived worlds.

Based on the material, the generating question was formulated. It starts by questioning how one of the most common drugs used in households can relieve patients from their pain, how the functioning can be described from a mathematical perspective and how that description can be used to design a correct dosing. The full formulation is shown below:

> Q_0: How can a patient be relieved from his pain by painkillers like paracetamol – how does deposit medication work and how can this be modelled mathematically? Q_1: Explain the biological functioning and consequences of

> taking paracetamol orally versus taking it intravenously. Q_2: Create a mathematical model using differential equations that illustrates the two processes and solve the equations in the general case. Q_3: Give a concrete example, where the patient is relieved from pain and estimate from your own model how often paracetamol has to be dosed – which parameters (absorption, elimination factor, bioavailability) are important to be aware of? $Q_{3,1}$: Does it make any difference whether the dose is given oral or intravenously? Use your models while giving your answer. (translated from Danish)

Notice that some of the derived questions are already given along with the generating question in order to guide the inquiry of the students (Chevallard, 2012, p. 11). It is crucial for the SRP to be successful that the students gets some guidance and are not left alone with a too open and overwhelming question. In this setting, the regulation of students' and teachers' work further necessitates that some of the "guiding" is provided from the outset. It should be possible for the students to see, from the outset, that their praxeological equipment in biology and mathematics can help them answer the generating question, and the given derived questions serve this purpose, asking for more specific cases to guide and delimit the student inquiry.

The formulation of the questions was followed by an a priori analysis before handing out the assignment. This a priori analysis will be presented in section on results.

Methodology

To carry out an *a priori* analysis means to explore what derived questions and answers could occur from the particular formulation of Q_0, i.e. what possible paths the students could follow based on their expected praxeological equipment and available media; concretely, a complete "tree" of derived questions and answers is produced. Figure 1 and 2 show the diagrams of the a priori analysis for the SRP considered in this study. In this case, the *a priori* analysis led to minor corrections of the design before it was tried out with students.

The school does not allow the use of lessons for guidance or classroom debate on the progress of the students work. Therefore, other ways to keep track of the students' work with the SRP were developed. To record the students' first thoughts on the generating question, they were asked to provide their spontaneous answer to the question in writing immediately after reading it. Two and four weeks later the students were asked to answer the following questions:

> What is your answer to the generating question right now? What have you done to answer the question? What are you planning to do next in order to come up with more fulfilled answers?

The teachers were only allowed to answer questions from the students after class. These conditions for guidance made it hard to track the exact progress of each student. Therefore the students were told only to ask questions if they gave them in writing by e-mail before meeting the teachers. It actually turned out that most questions could also be answered by e-mail. Examples of questions are given in the section on results.

Because of the little data available from the students working process, it is the outcome of the students' writings which is the main evidence of their study and research process. The reports were analysed as SRP, using the method developed earlier (cf. Hansen, 2009, pp. 60) and which I now describe. While reading the reports every small section was identified with the (derived) question it treats. An example could be "how to model a one-compartment system when knowing the diffusion of the drug from the vein alone relies on the elimination factor?" This can be answered by the praxeology of "setting up a first order differential equation from given conditions". This is a praxeology on mathematical modelling using differential equations. In this way the entire report was split up in small pieces of questions and answers (Q_i, A_i). The organisation and relation between praxeologies can be depicted by tree diagrams (see figure 3). The relations were identified from the way the student referred to or drew on previously presented praxeologies ie. sections or part of sections. When it comes to the parts of the reports consisting of pure biology, the praxeologies were only identified as a question and the answer given by the student – that is, I did not model or analyse biological praxeologies in detail, due to lack of knowledge in the field of biology.

The analysis of the students' reports was compared to the a priori analysis. The comparison of the diagrams showed to what extent the students had developed the intended praxeologies and maybe some unexpected ones. At the same time the diagrams show to what extent the two disciplines were incorporated and combined in the report and solutions. This helps to answer the crucial question: Does the formulation of the generating question function as a bidisciplinary task and do the student use and combine both disciplines while answering the assignment?

For the last part of the project the students were told to continue to ask questions by e-mail while rewriting their reports. The synopses were handed in electronically and during the oral exam written notes were taken. From this the new praxeologies were identified even though the synopsis format is not suitable for a thorough praxeological tree diagram analysis. Through these steps of analysis the results of the design and the students activities can be presented.

Results

As expected, there was a great diversity in the students' reports. Some students worked thoroughly with the questions and were able to formulate derived questions themselves – even explicitly. Others were not able to see the use of mathematics in the assignments and tried to answer the exact questions formally, without further inquiry. This was expected as the class was not particularly "strong" (mathematically and academically) – but many of them were hard working and for them the study phase seemed very enriching. They clearly developed new mathematical praxeologies during their work with the SRP, as will be explained in detail in this section

The analysis of the formulation of the assignments gives the tree diagram figure 1 which shows the connections between the generating question and derived ones.

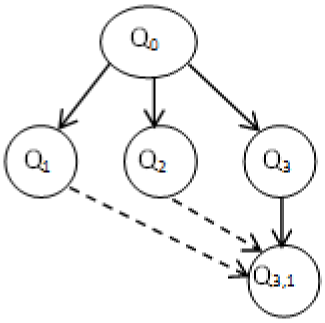

Figure 1: Tree diagram showing the formulation of the assignment

The dotted line indicates that question number $Q_{3,1}$ draws on the knowledge worked out as answers to questions Q_1 and Q_2. The solid lines indicates that the questions are derived questions in the sense described by Chevallard (2006); in short, derived questions are natural prolonging of the former in order to achieve a more detailed inquiry. The tree diagram in figure 1 is part of the *a priori* analysis of the assignment. To get a more complete picture of the potentials of the SRP design, a full *a priori* analysis was made. This analysis is presented in the tree diagram of figure 2. Question numbers refer to the same as those in figure 1. The rest of the questions are derived questions, which are the questions students are intended to work with in this particular SRP. The answers to those questions are the praxeologies the students are supposed to develop in the field of differential equations and nerve physiology in relation to the diffusion of a drug in the body. The lines connecting the questions have the same interpretation as in figure 1.

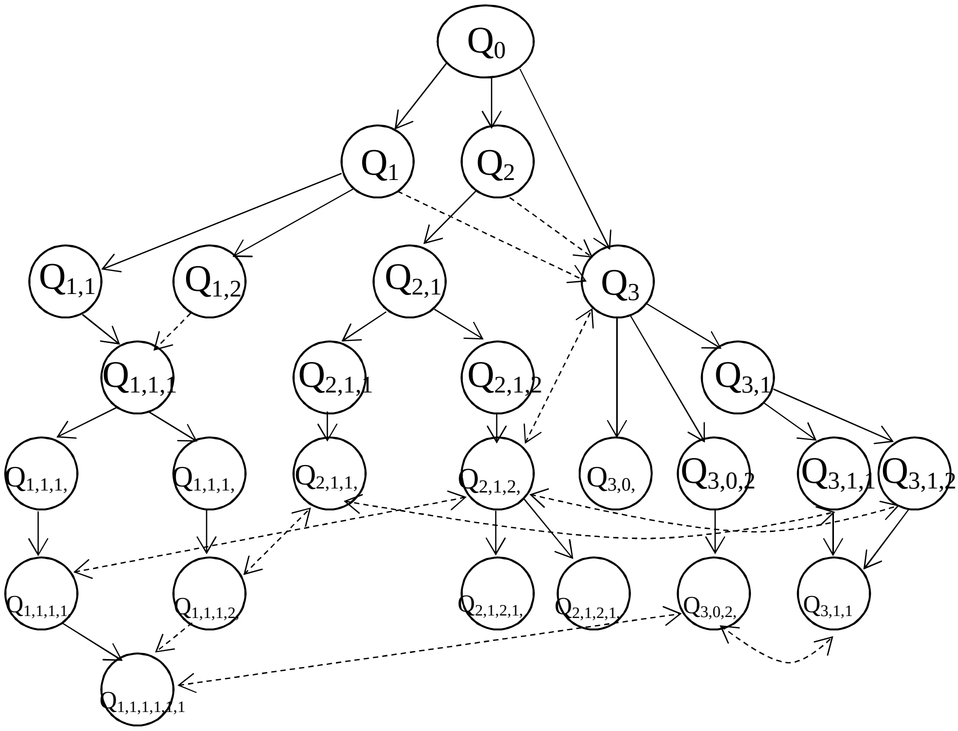

Figure 2: Tree diagram of the a priori analysis of the assignment. See the text below for the contents of each question.

The questions formulated by the author[3], only having an academic background in mathematics, during the a priori analysis is the following where question numbers corresponds to those of figure 2. The questions representing the expected praxeologies are written in italic (these are not handed out to the students):

Q_0: How can a patient be relieved from pain, using a drug like paracetamol? How does deposit medication work and how can this be modelled mathematically?

Q_1: Explain the biological functioning and consequences of taking paracetamol orally versus intravenously.

$Q_{1,1}$: What is the biological mechanism underlying the concept of pain?

$Q_{1,2}$: What kind of drug is paracetamol?

$Q_{1,1,1}$: How does paracetamol function in the body?

[3] As mentioned earlier, this research was conducted by the author, who is the mathematics teacher and a didactic researcher. This means that the perspective of the questions is considered only from the standpoint of the mathematician. The *a priori* analysis would look differently, if it was carried out by others, with a different academic background.

$Q_{1,1,1,1}$: How does paracetamol function when it is dosed orally?
$Q_{1,1,1,1,1}$: How is paracetamol transported from the stomach to the vein biochemically seen?
$Q_{1,1,1,1,1,1}$: How long does it take from the drug is injected in the vein till a person is relieved from his pain?
$Q_{1,1,1,2}$: How does paracetamol function when it is dosed intravenously?
$Q_{1,1,1,2,1}$: What is the biochemical functioning of paracetamol in the vein?

Q_2: Set up a mathematical model using differential equations that illustrates the two processes and solve them in the general case.
$Q_{2,1}$: What is a differential equation?
$Q_{2,1,1}$: What can the differential equation $y' = ky$ model and what is the general solution?
$Q_{2,1,1,1}$: How do we model a one compartment system modelled using the elimination factor?
$Q_{2,1,2}$: What can the differential equation $y'(t) = c_1 z(t) - c_2 y(t)$ model and what is the general solution?
$Q_{2,1,2,1}$: How can we model the effects of the absorption using differential equations?
$Q_{2,1,2,2}$: How can we model the effects of the bioavailability using differential equations?

Q_3: Give a concrete example, where the patient is relieved from pain and estimate from your own model how often paracetamol has to be dosed – which parameters (absorptivity, elimination factor, bioavailability) are important to notice?
$Q_{3,0,1}$: What numbers can be put on the relevant notions and what do they tell?
$Q_{3,0,2}$: How can we model multiple dosing using the existing models?
$Q_{3,0,2,1}$: How often must the doses be given in order for the patient not to feel any pain?
$Q_{3,1}$: Does it make any difference whether the dose is given oral or intravenously? Use your models to support your answer.
$Q_{3,1,1}$: What does the model of multiple dosing look like in the case of intravenous dosing?
$Q_{3,1,2}$: What does the model of multiple dosing look like in the case of oral dosing?
$Q_{3,1,1,1}$: What differences appear while comparing the graphic presentation of the two functions of multiple dosing?

The diagram of figure 2 is satisfactory in terms of the requirements for the design, as it shows several paths for the students to pursue, with possibilities for the students to work interdisciplinarily, to activate their initial praxeological equipment, and potentially to develop new praxeologies in the field of differential equations and nervous physiology.

Results of the students writings

I will now present the outcomes of this teaching design. I will do so by presenting a well written report richly unfolding the intended praxeologies. After this I present a report written by a weak student only poorly unfolding the potentials of the design and finally I give some of the outcomes of the synopsis and oral exam.

As mentioned some of the students were able to realise these potentials and wrote mathematically rich and substantially bi-disciplinary reports. Figure 3 shows a tree diagram of the analysis of one of these reports in its final state. We can say a little about the process of the author of this report from what she wrote as spontaneous and intermediate responses to the generating question. Just after seeing the question, she noted that she needed to know something about the dosing of the drug in relation to the weight of a given person. She calls it the “strength” of the drug. And she needs to know something about how long the drug stays in the body, and refers to what she calls “the half-life of the drug”[4]. This she planned to use to find out how to relieve a patient from pain for a longer time period. This indicates that she believed from the start that the model involves an exponential function, without knowing anything else about this question.

Two weeks later (when again asked for her ideas on the generating question), this student also wants to know more about how paracetamol is functioning biologically, and she indicates that she needs more knowledge on mathematical modelling. This is what she is planning to study the next weeks. This indicates that she is narrowing down to more specific questions for her to answer.

The notion model or modeling in the students writings probably refer to the one the student encounters in her textbook and official documents for Danish high school, which is somehow close to the notion in mathematical competence theory (see Niss et al., 2002 and Blum & Fermi, 2009, p 46). However the approach to modeling in ATD is that it is the development of praxeologies in two domains answering a generating question.

In the text below several technical terms are used. They are translations of the notions the student used. Many of them comes from the biological field being modelled and therefore will not be explained further. As to differ questions formulated by the student from those she has adopted from the assignment handed out, the students’ questions and formulations are put into squared brackets.

[4] She knows this notion from previous work on exponential function and from radio activity.

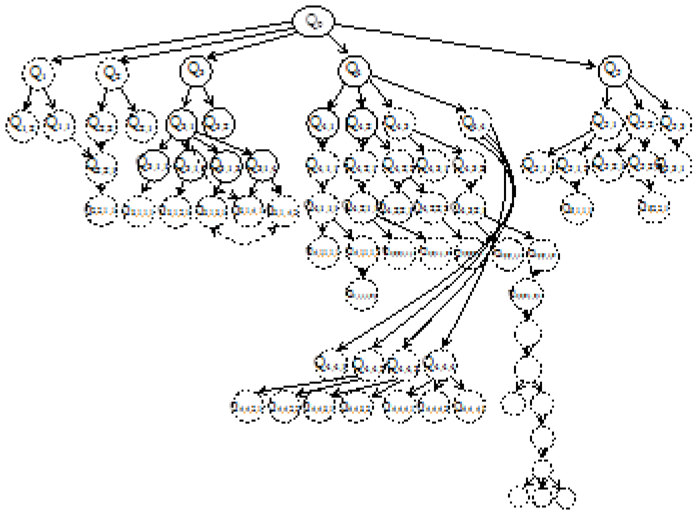

Figure 3: Tree diagram of the analysis of a handed in report. Question contents are detailed in the text.

This student actually formulated a number of derived questions in her report and used them as headings, e.g.: “Q_1: How is pain registered? Q_2 How does paracetamol relieve pain (pharmaco dynamic)? Q_4: How can the dosing be modelled mathematically based on the biological knowledge?” (Appendix A) Other headings were not phrased as questions but were simply a word as “Absorption”. Derived questions not phrased as above are identified through further discourse analysis of the text. Examples are $Q_{2,1}$: How does paracetamol relieve pain relative to the amount of dose? And $Q_{2,2}$: How does paracetamol relieve diffuse pain. The student relates her answer to this question to $Q_{1,1}$: How are diffuse pains registered and what are diffuse pains? This is indicated in figure 3 by a dotted line. I will now give a short review of this report, for an extensive list of the questions the student treats see Appendix A.

The student starts by posing and answering the questions: “How is pain registered?” (Q_1) and divides this into the treatment of what are diffuse pains and how they are registered as well as what are diffuse pains and how they are registered ($Q_{1,1}$ and $Q_{1,2}$). Then she poses the question: How does paracetamol relieve pain (pharmacodynamic)? (Q_2). This is dealt with through questioning how paracetamol relieves pain relative to the amount of dose, how it relieves diffuse pain, what effect the drug has on the nervous system and what is known about the drug in general ($Q_{2,1}$, $Q_{2,2}$, $Q_{2,1,1}$ and $Q_{2,1,1,1}$).

After this the student poses the question: How is paracetamol transported through the body (pharmacokinetics)? (Q_3). This is investigated through the study of how drug is transported in the case of orally dosing, how drug is transported in the case of intravenous dosing. The former is further explored by showing how the drug is absorbed in the body, how this process runs in the small intestine and how the drug is distributed in the body, which leads to a description of the biochemical conditions and mechanisms that are relevant for this problem, hence how substances are transported through cell membranes ($Q_{3,1}$, $Q_{3,2}$, $Q_{3,1,1,1}$, $Q_{3,1,2}$, $Q_{3,1,2,1}$ and $Q_{3,1,2,1,1}$). Finally the metabolism of paracetamol including the chemical reactions occurring and the elimination with the role of the kidneys and timescale of the process is presented ($Q_{3,1,3}$, $Q_{3,1,3,1}$, $Q_{3,1,4}$, $Q_{3,1,4,1}$ and $Q_{3,1,4,2}$). These questions and answers all represent pure biological praxeologies, which are found relevant in order to model the processes of dosing paracetamol a long with the discussion of how should be used….

Next the student poses question number Q_4: "How can the dosing be modelled mathematically based on the biological knowledge?". She finds the answer by looking at the form of the model in the case of intravenous dosing, how the proportionality between added amount of paracetamol and elimination can be modelled, what can be described by the equation $\frac{\mathrm{dA}}{\mathrm{d}t} = -\mathrm{K}.\mathrm{A}$, the biological interpretation of –K with respect to the former treated questions ($Q_{4,1}$, $Q_{4,1,1}$, $Q_{4,1,1,1}$ and $Q_{4,1,1,1,1}$). These are all bidisciplinary praxeologies where the student alternates between using the established biological praxeologies in the construction and justification of a first order differential equation – a mathematical object. She ends this section by finding the complete solution using a CAS tool and showing by hand, that this solution actually solves the equation ($Q_{4,1,1,1,2}$ and $Q_{4,1,1,1,2,1}$). These two questions are identified as pure mathematical.

After treating the more simple case she looks at the oral case and performs the same praxeologies though taking into account that she needs to treat the two compartments separately and combine these results in one equation describing the entire system ($Q_{4,2}$, $Q_{4,2,1}$, $Q_{4,2,1,1,}$ $Q_{4,2,1,1,2}$ and $Q_{4,2,2}$). She further argues how the added amount of paracetamol can be described by the solution to the differential equation of the stomach compartment and how the model incorporates the bioavailability ($Q_{4,2,2,1}$ and $Q_{4,2,2,2}$). Again, this is denoted bidisciplinary praxeologies. The student investigates what can be described by the equation: $\frac{\mathrm{dA}}{\mathrm{d}t} = \mathrm{A}.\mathrm{K}_a.\mathrm{A}^{stomach} - \mathrm{K}.\mathrm{A}$, finds the solution and argues that the model solves the equation as in the simple case ($Q_{4,2,2,2,1}$, $Q_{4,2,2,2,1,1}$ and $Q_{4,2,2,2,1,1,1}$).

The student uses the two models to form functions describing the concentration of paracetamol in the blood, she gives all parameters numerical values and discusses both the mathematical and the biological interpretation of $K_a > K$ ($Q_{4,3}$, $Q_{4,3,1}$, $Q_{4,3,2}$,

$Q_{4,3,2,1}$ and $Q_{4,3,2,1,1}$). Finally she discusses the knowledge of the numeric versions of the functions and its graphical representation. From these representations she discusses the long term effects, high amount dosing and how patients can be relieved from their pain through multiple dosing and how this can be carried out repeated dosing with constant amount of paracetamol ($Q_{4,3,2,1,2}$, $Q_{4,3,2,1,2,1}$, $Q_{4,3,2,1,2,1,1}$, $Q_{4,3,2,1,2,1,1,1}$ and $Q_{4,3,2,1,2,1,1,1,1}$). The method of these praxeologies is mainly mathematical but constantly links to her knowledge in the biological field and she concludes on the biological issues from the mathematical models. Hence These praxeologies are regarded bidisciplinary. The student further notes that multiple dosing leads to a concentration alternating around a mean called steady state. She uses the mathematical models to determine steady state level and whereas the patient feel a constant relieve of pain when maximum recommended dose is given every 4 and 6 hours (the two standard time intervals) ($Q_{4,3,2,1,2,1,1,1,2}$, $Q_{4,3,2,1,2,1,1,1,2,1}$, $Q_{4,3,2,1,2,1,1,1,2,1,1}$, $Q_{4,3,2,1,2,1,1,1,2,1,1,1}$, $Q_{4,3,2,1,2,1,1,1,2,1,1,2}$ and $Q_{4,3,2,1,2,1,1,1,2,1,1,3}$). As the praxeologies just mentioned these are regarded bidsciplinary for the same reasons.

In the end the student compares the two ways of dosing the drug with respect the type of pain it is supposed to relieve. This is done by a comparing the concentration profiles, discussing similarities and differences (Q_5, $Q_{5,1}$, $Q_{5,1,1}$, $Q_{5,1,2}$ and $Q_{5,1,1,1}$). These praxeologies are likewise bidsciplinary since biological results are based on mathematical models treated by mathematical tools. The treatment of Q_5 ends in a further investigation of intravenous dosing, as to what kind of situations and what kind of lack in health condition among patients calls for this kind of dosing ($Q_{5,2}$, $Q_{5,2,1,1}$, $Q_{5,2,1,2}$ and $Q_{5,2,1,2,1}$). These praxeologies are mainly biological. They discusses some of the results showed in the graphical representations of the concentration function, but it is only treated in a biological context. The last two biological praxeologies performed are examining the relation between concentration functions and the recommendations on the painkiller packages and further discusses whether the functions implies a change of recommendations ($Q_{5,3}$ and $Q_{5,3,1}$).

After this the students returns to the models and functions she has created discussing the limitations of these ($Q_{4,4}$ – the choice of numbering reflects praxeologies relation the rest of the SRP and not the chronology of the report). She starts by discussing in general terms the meaning of modelling the real world, then she turns to biological conditions effecting absorption, bioavailability and the pharmacokinetics in general due to the patient being pregnant, a child or elderly. This is supported by listing the consequences of taking other drugs, eating, vomiting or having diarrhea while taking paracetamol ($Q_{4,4,1}$, $Q_{4,4,2}$, $Q_{4,4,2,1}$, $Q_{4,4,2,2}$, $Q_{4,4,3}$, $Q_{4,4,3,1}$, $Q_{4,4,3,2}$, $Q_{4,4,4}$, $Q_{4,4,4,1}$, $Q_{4,4,4,2}$ and $Q_{4,4,4,3}$). These praxeologies are mainly biological though they are all used in a critique of the models created by the student.

As indicated, the path starting from Q_4 is mainly treating the mathematical organisation. The answers are constantly referring to the biologically field which is being modelled. Still the student uses pure mathematically praxeologies such as $Q_{4,1,1,1,2}$ and $Q_{4,1,1,1,2,1}$. These praxeologies are examples of intended mathematical praxeologies which the student has developed working with this specific SRP.

Comparing figure 2 and 3 it is obvious that the student has followed most of the intended path and even added necessary details in order to answer the question in a satisfying manor. The student also adds branches not intended such as $Q_{4,3,2,1,2,1,1,1,2}$, where she treats the notion of steady state concentration both mathematically and biologically.

The student asks three questions during the writing process and they concerns her critique of the mathematical models – she lists 3 points and asks if they are reasonable – the notion of deposit medication and how it is interpreted and finally she asks if she can put her mathematical calculations in appendix due to many pages of text. This means that her study of the sources is done without help from the teachers and the tree diagram is showing her working process with the SRP. This diagram and others like it (based on other student reports) show that it is possible to create bidisciplinary assignments on the basis of SRP that function well for some students.

Rich outcomes were found in other reports as well. Students normally having difficulties working on the theoretical level engaged themselves in the SRP and managed to develop arguments on how to model the transportation of a drug in the vein. One student explains that when you are modelling the change of the amount of drug in the vein, differential equations are suitable since they model how fast something changes. In a particular case she needs to know how much drug is added and how fast it eliminates from the vein. From this she presents the model, with the factors representing added and eliminated amount of drug. This student is normally quick at solving simple standard tasks, but she rarely argues precisely at the theoretical level. The reason for the change in the setting of the SRP could be that the student consulted classmates and was inspired by their work. Another reason could be that the entire assignment makes it obvious for her that she needs to justify her model explicitly – it is not possible to answer the questions "mechanically".

The students having difficulties to engage seriously with the SRP were those who generally find mathematics and biology hard. Some of those students did not find the topic interesting. They were able to solve simple questions involving simple praxeologies. Some of them did not succeed to combine mathematical and biological praxeologies, these students mainly referring the source (Jonsdottír et al., 2009) and some textbooks on the biological topic. When they were supposed to interpret the models, they would invent two persons in order to compare the amount of drug in the bodies – comparing a child and an adult, and, ignoring that the biological factors are different from children to adults. This shows that they were merely able to study the handouts based on separate praxeologies already developed during mathematics and biology classes. They did not develop the intended new praxeologies and so they were only able to solve simple tasks in the field of differential equations and human physiology. An example of a tree diagram of a report handed in by one of the weak students is shown in Figure 4.

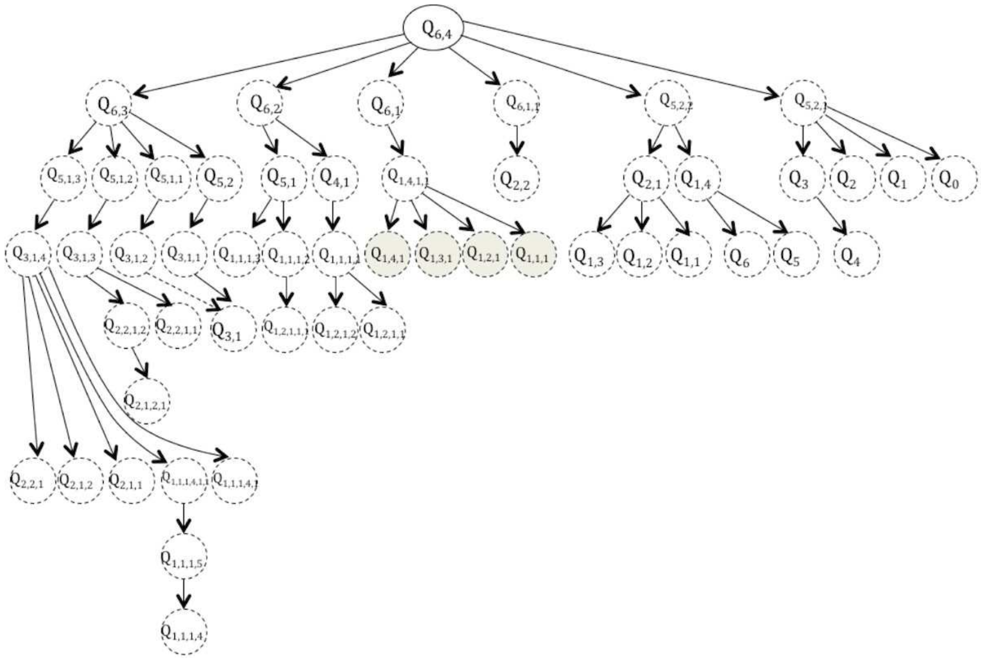

Figure 4: Tree diagram of the analysis of a report handed in by a weak student. An outline of the questions is presented in the text

The diagram of Figure 4 shows that the student spread her attention on many different directions (subquestions) but none of the questions are treated thoroughly or connected to others. The path starting from Q_6 is the only one which involves mathematics. The student presents some equations for calculating the amount of drug in the vein of an “average person”, the maximum concentration of drug in the case of intravenous dosing, the time it takes to reach maximum concentration and finally an equation of the steady state concentration. She does not mention differential equations at all or how to deduce the models from them. This implies that the student has not developed the intended mathematical praxeologies. The same goes for biology. The presentation of the biological answers is very superficial and the text only cites sources in general terms. The praxeologies coloured grey in figure 4 actually short versions of question number $Q_{1,1}$, $Q_{1,2}$, $Q_{1,3}$,

$Q_{1,4}$ – nothing new is added to the text book presentation of the notions of absorption, distribution, metabolism and elimination. Only once the student combines two answers ($Q_{1,3,1}$ and $Q_{1,4,1,1}$ on how the kidneys contribute to the elimination of the drug). The rest of the report had a parallel structure, which indicates that the student was not able to combine the different answers to subquestions. The teacher proposed, as an explanation, that this student is often doing her work in last minute, and so she did not see how much effort she had to invest to properly answer the questions. This suggests that working with SRP requires adaptation through more than one experience, at least for some students.

Other students simply were not able to see the relevance of mathematics in the response to the generating question. An example is a student who answer the question on modelling by citing the handout without commenting or using the model. This indicates that the student only sees this question as a way to add mathematics to the report or project, but not as something necessary from the theoretical point of view. She spends several pages on drug development[5] and obviously finds this interesting. Maybe she did not have enough time for the mathematical part because she spent her time on what she found most interesting. This student usually was able to combine simple praxeologies but was not theoretically strong. This supports the hypothesis that the student did not see the need of mathematics to answer the generating question. How to deal with this concern will be discussed later.

Outcomes of the synopses and oral exam

To begin with, the focus was put on the reports, but interesting findings occurred during the students' work with synopses and at the oral exam. The students who did well on the reports were still performing well in the synopsis and at the oral exam. Some students who made acceptable reports were able to improve their work after the written feedback. As mentioned earlier I did not get equivalently systematic evidence from this part of the students' work. The findings presented below are therefore simple and tentative descriptions of student work in this phase.

One of the most interesting observations occurred with a student who had made a nice report using differential equations and explaining them using knowledge from biology. When asked to place the case of using paracetamol in a broader context she did an Internet search and found articles written by Danish researchers on the use of the drug during pregnancy. The article discussed whether there was a significant amount of degeneration of the genitals of baby boys when the mothers had taken paracetamol during pregnancy. The result was not clear and in fact there

[5] As part of the teaching the class visited Faculty of Pharmaceutical Sciences at University of Copenhagen to learn about drug development research and how drugs are distributed and functioning in the body.

is no recommendation against the drug during pregnancy today. The article gave the numbers of women tested, the expected percentage of degenerations and the actual number of boys born with these problems. The researchers used a statistical test with level of significance at 5 %. The student did not know the particular type test therefor she performed a χ^2-test instead, which gave a p-value just above the level of significance. She used this in a discussion of the recommendations on whether the drug should be available outside pharmacies. She further referred to articles found in journals and on the Internet. This study was very surprising for both teachers. They did not know the relation to pregnancy nor had χ^2-test been part of the intended mathematical praxeologies for the SRP, but it was a tool the student knew from classes and put to use in a new context. This is a nice example of a potential of SRP: "that the contents learnt […] have not been planned in advance" (Chevallard, 2012, p. 7).

The student who did the report represented in Figure 3 continued her work on the effect of paracetamol in the brain and the nervous system. She was able to explain how new ideas could be modelled and tested, as she focused on the problematique of mentally ill people whose abuse of paracetamol cause long-term damages. She discussed this in relation to question of the drug being sold legally outside pharmacies.

The results mentioned above from the oral exam are examples of students combining the study of works of others combined with an autonomous treatment of results. In this sense a more general aim for the SRP was reached. On the other hand, the students who handed in poorer reports were not able to improve for the synopsis and did not perform well at the oral exam either. There remains, thus, a considerable challenge in making this SRP successful for all students.

Discussion

Many students engaged in a real study process, to find answers on their own rather than just citing the works of others, which on the other hand seems to be the pitfall for other students. The real world problem seems to motivate the students for an inquiry where they can use and combine their previous knowledge and experience from both mathematics and biology.

The SRP enables most students to make the two disciplines interact. As already said it is crucial to choose a strong generating question that engages the students to develop the intended praxeologies, and the quality of this choice could secure the possibility of actual interdisciplinary work. This means that a thorough *a priori* analysis must be the starting point of all bidisciplinary SRP designs since the interaction between disciplines is clearly not obvious or automatic.

But there are still issues to deal with if SRP should be successful for all students. The interplay between the two disciplines was weak or absent in the work of some students. These students fail to see the need of one discipline (primarily mathematics) or were not able to realise it in the given setting. Probably it requires more directions, by way of concise questions in both disciplines, to secure that students develop new intended praxeologies. This was seen in the report written by the student focusing on drug development as well as the report depicted in Figure 4. The big question is how to detect and treat these obstacles while creating the design. This relates to the *a priori* analysis of the SRP designs and to a more theoretical study of the possible interplays of mathematics and biology. Is it sufficient that two teachers (representing each discipline) formulate the design? or is it necessary for the teachers to do an analysis of the didactic transposition (e.g. see Bosch & Gascón, 2006, pp. 55) of the interplay of the involved scientific disciplines in order to identify interdisciplinary praxeologies combining the school disciplines? What are the scientific interactions between biology and mathematics and how can they be transposed to interactions between the secondary school subjects? To identify bidisciplinary praxeologies and what questions they answer we need to know more about what a biological praxeology is (and more generally, what are praxeologies in the natural sciences). This is formulated by Mortensen (2011) and Madsen & Winsløw (2009) but in other contexts.

Another approach to bidisciplinarity is found by Hansen (2009, p. 35) who suggests that what constitutes a discipline (as well as interdisciplinary praxeologies) is the methods of the disciplines used in the particular praxeology together with the objects of knowledge. This means that in order to formulate more concise questions, it is needed to identify the methods of mathematics and biology respectively as well as the relevant objects of knowledge. From this one can form the didactic transposition of the bidisciplinary knowledge, which can be used in a reference model for the SRP while carrying out the a priori analysis. In this way one might be able to create the more concise bidisciplinary questions which seem to be needed by some students.

The general hypothesis is that after identifying the possible interdisciplinary praxeologies, one will be able to formulate more exact questions which allow students to see the need of combining the two disciplines, and to develop more precise and complete answers. Also, by focusing on the interplay between the disciplines we might be able to make the students develop new monodisciplinary (e.g. mathematical) praxeologies.

Another concern regarding the students who wrote the poor reports is if the generating question hinders their engagement. It is obvious it is almost impossible to find generating questions which everybody finds equally exiting. Maybe the question seemed too vague compared to what they are used to. This obstacle can be handled using SRP in every day teaching so the students know the concept and what is required of them.

Knowing how students generate these new or derived questions would certainly be another way to overcome this challenge formulating good generating questions. It is an open problem in ATD. It is assumed that if posed a generating question within reach of the students existing praxeological equipment they are able to consult relevant medias – or in this study they know, that they need to study more advanced differential equations or exponential models. Therefor they consult medias on these topics and from the media pose new more concrete questions. It is assumed if the generating question is more guided in order to secure the student develop certain praxeologies, some of the potentials of the design and inquiry process disappear. This is also discussed in relation to inquiry in (Artigue & Blomhøj, 2013, 806). Further study in this matter could be interesting to pursue.

Some of the difficulties among the weak students might have been avoided, if the external conditions and constraints had been different. In the study of Barquero, Bosch and Gascón (2007) and Thrane (2009) the procedure of carrying out the SRP is that students share their findings. They present their findings and discuss academically what path tends to be the most promising one, and then everybody follows it. These sequences secure that no one remains stuck, with no ideas of how to progress. There are several reasons for organising the SRP process this way. When the student argues that one praxeology is a better or more general solution to a certain task they learn the scope and limitations of each praxeology, which helps them developing the intended knowledge.

The reason for not creating these sessions during the testing of the teaching design was that the requirements set by the institutional frame prescribed that the project should not use mathematics or biology lessons for the work. The students were supposed to work autonomously or in groups of two – not as a whole class together. This condition makes sense since they are supposed to get training for their final autonomous project. But the students did actually meet after classes to discuss their findings. This process seemed fruitful. Still some of the students who needed it the most did not attend. Because of this one could argue for a loosening of the constraints so that it is allowed for the teacher to organise such sessions and to guide the debate. If the students engaged themselves in this process one could argue that they still work autonomously – just in a more collective manner.

A final point: for the bidisciplinary assignments to function, the teachers must engage themselves in what could be called a bidisciplinary SRP for themselves as well. It is not evident that both teachers know the knowledge field of the other discipline. Therefore, in order to form questions concerning the interplay between the disciplines, the teachers must study a certain amount of the other discipline. For an academically trained person, this task is reasonable and crucial for the SRP to function as bidisciplinary assignment. The gained knowledge should be used to perform the a priori analysis and reveal the possibilities and limitations of the two disciplines in treating a given problematique or generating question.

Conclusion

The experiment and open issues with the SRP design showed clear evidence for the advantages of using SRP as a model for designing bidisciplinary assignments. The *a priori* analysis secures that the possible paths of inquiry are connected in the sense that the disciplines are interacting – not just in theory but also in reality. The reports the students handed in substantiated this finding since most students actually pursued the intended paths and even identified new directions, corresponding to substantial new derived questions. The students even succeeded in giving more detailed arguments and rich mathematical sections of their reports. Still the format of an academic-like autonomous written report is a difficult task for the students, therefor it is suggested that students encounter these types of reports more often in order to deliver rich and detailed documentation for their inquiry process, which these SRP's represents.

The experiment also showed that the teachers must be prepared to engage themselves in a SRP as well. For the teacher to carry out the *a priori* analysis she must cross disciplinary boundaries in order to see possibilities and pitfalls in the SRP design. The teachers must do the inquiry of the bidisciplinary field before formulating the assignment. Though it should be noted that boundaries between mathematics and biology are historical and evolving constructions that do not have to be taken from granted outside school institutions – nor in the praxeological analysis done here in the case of mathematical questions extended to biological phenomena treated in the SRP.

Moreover the tree diagrams shows to be a strong tool for depicting the praxeologies presented in the reports as the result of the discourse analysis. This diagram compared with the one from the a priori analysis gives a more clear view to what extend the intended praxeologies are present in students work. Concretely the two presented tree diagrams show two very different reports. It could be a question for further study to what extend the tree diagrams can be direct indicators for the richness of students writings.

The experiment suggests that some of the conditions for carrying out this particular design were not to the advantage of all students. The fact that almost all work on the SRP was placed outside school, and the lack of debate on particular paths to take during the inquiry, were problematic to some students. On the other hand many students were successful in engaging themselves with the SRP.

The experiment finally revealed questions for further inquiry. It is still an unresolved task to formulate bidisciplinary questions which all students see as such. Moreover the notion of bidisciplinary praxeology needs further exploration in terms of how to define and identify them, and in terms of their role for students' success with monodisciplinary praxeologies. Further it is suggested that in order to carry out a sufficient a priori analysis it would be enriching to formulate an reference epistemological model as described in the didactic transposition. It is supposed that this could enlighten some disconnection regarding the students inability to see the full need of mathematics in their answer to the generating question.

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Appendix A

The entire report treats the questions listed chronologically in Appendix A. The question numbers refer to those of figure 3:

Q_1: How is pain registered?
$Q_{1,1}$: “What are diffuse pains and how are they registered?”
$Q_{1,2}$: What are acute pains and how are they registered?
Q_2: How does paracetamol relieve pain (pharmacodynamic)?
$Q_{2,1}$: How does paracetamol relieve pain relative to the amount of dose?
$Q_{2,2}$: How does paracetamol relieve diffuse pain?
$Q_{2,1,1}$:What is known about the effect of paracetamol on the nervous system?
$Q_{2,1,1,1}$: What is known about paracetamol in general?
Q_3: How is paracetamol transported through the body (pharmacokinetics)?
$Q_{3,1}$: How is the drug transported in the case of orally dosing?
$Q_{3,2}$: How is the drug transported in the case of intravenous dosing?
$Q_{3,1,1}$: How is the drug absorbed in the body?
$Q_{3,1,1,1}$ How does this process function in the small intestine?
$Q_{3,1,2}$: How is the drug distributed in the body?
$Q_{3,1,2,1}$: What biochemical conditions and mechanisms are relevant for this process?
$Q_{3,1,2,1,1}$: How are substances transported through cell membranes?
$Q_{3,1,3}$: How is paracetamol metabolized?
$Q_{3,1,3,1}$: Which chemical reactions occur during the metabolism of paracetamol?
$Q_{3,1,4}$: How is paracetamol eliminated in the body?
$Q_{3,1,4,1}$: What is the role of the kidneys, with respect to the metabolites?
$Q_{3,1,4,2}$: What is the timescale or half-life of paracetamol in the body?
Q_4: How can the dosing be modelled mathematically based on the biological knowledge? $Q_{4,1}$: What does the model look like in the case of intravenous dosing?
$Q_{4,1,1}$: How can the proportionality between added amount of paracetamol and the elimination be modeled?
$Q_{4,1,1,1}$: What is described in by the equation $\frac{dA}{dt} = -k \cdot A$?
$Q_{4,1,1,1,1}$: What is the biological interpretation of –k?
$Q_{4,1,1,1,2}$: What is the complete solution to the differential equation?
$Q_{4,1,1,1,2,1}$: How can one check the validity of a given solution?
$Q_{4,2}$: What does the model look like in the case of oral dosing with a two-compartment system?
$Q_{4,2,1}$: How can the stomach compartment be modeled?
$Q_{4,2,1,1}$: What is described by the equation $\frac{dA^{mave}}{dt} = -k_a \cdot A^{mave}$?
$Q_{4,2,1,1,1}$: What is described by $-k_a$?

$Q_{4,2,1,1,2}$: What is the solution to the differential equation?
$Q_{4,2,2}$: How can the dosing be modeled from the perspective of the vein compartment?
$Q_{4,2,2,1}$: How can it be argued that the added amount of paracetamol can be described by the solution to the differential equation of the stomach compartment?
$Q_{4,2,2,2}$: How is the bioavailability incorporated in the model?
$Q_{4,2,2,2,1}$: What is described by the equation: $\frac{dA}{dt} = F \cdot k_a \cdot A^{mave} - k \cdot A$?
$Q_{4,2,2,2,1,1}$: What is the complete solution to the differential equation?
$Q_{4,2,2,2,1,1,1}$: How can one check the validity of a given solution?
$Q_{4,3}$: How can the concentration of paracetamol in the blood be modeled?
$Q_{4,3,1}$: How does this look in the case of intravenous dosing?
$Q_{4,3,2}$: How does this look in the case of oral dosing?
$Q_{4,3,2,1}$: What numbers are reasonable for the constants: K, F, F_a and V?
$Q_{4,3,2,1,1}$: What is the biological interpretation of K_a>K?
$Q_{4,3,2,1,2}$: What function describes the concentration?
$Q_{4,3,2,1,2,1}$: How does the function look graphically?
$Q_{4,3,2,1,2,1,1}$: How can the concentration be interpreted in relation to longtime effect and high amount of paracetamol?
$Q_{4,3,2,1,2,1,1,1}$: How can a patient be relieved from his pain due to multiple dosing?
$Q_{4,3,2,1,2,1,1,1,1}$: How can this be carried out sequentially with constant amount of paracetamol?
$Q_{4,3,2,1,2,1,1,1,2}$: What is steady state?
$Q_{4,3,2,1,2,1,1,1,2,1}$: When and how is this state reached?
$Q_{4,3,2,1,2,1,1,1,2,1,1}$: What is concentration at steady state?
$Q_{4,3,2,1,2,1,1,1,2,1,1,1}$: In which cases are the amount of dose 1000mg every 4 hours?
$Q_{4,3,2,1,2,1,1,1,2,1,1,2}$: In which cases are the amount of dose 1000mg every 6 hours?
$Q_{4,3,2,1,2,1,1,1,2,1,1,3}$: Which function is modeling multiple dosing?
Q_5: When and why is orally and intravenous dosing used respectively?
$Q_{5,1}$: How can the two concentration profiles be compared?
$Q_{5,1,1}$: When do the two profiles reach their maximum concentrations?
$Q_{5,1,2}$: When does the effect of paracetamol die out?
$Q_{5,1,1,1}$: When is there an effective difference between the two forms of dosing?
$Q_{5,2}$: When is intravenous dosing preferable?
$Q_{5,2,1,1}$: In which cases will time be the determining factor for choosing intravenous dosing?
$Q_{5,2,1,2}$: Under what health conditions are the intravenous dosing preferable?
$Q_{5,2,1,2,1}$: What kind of conditions of the stomach makes the intravenous dosing preferable?
$Q_{5,3}$: What is the dosing profiles telling about the dosing of paracetamol compared to the recommendations on the painkiller packages?

$Q_{5,3,1}$: How should paracetamol be dosed according to the profiles?
$Q_{4,4}$: What biological factors are disregarded in the mathematic models?
$Q_{4,4,1}$: What is the relation between a (mathematical) model and the real world?
$Q_{4,4,2}$: What other biological factors affect the absorption K_a?
$Q_{4,4,2,1}$: What effect causes other drugs taken a long with paracetamol?
$Q_{4,4,2,2}$: What effects are caused by eating while taking paracetamol?
$Q_{4,4,3}$: What factors can effect the bioavailability?
$Q_{4,4,3,1}$: What are the consequences of vomiting?
$Q_{4,4,3,2}$: What are the consequences of diarrhea?
$Q_{4,4,4}$: What other factors affect the pharmacokinetics?
$Q_{4,4,4,1}$: What effects are caused by pregnancy?
$Q_{4,4,4,2}$: What effects are due to the person being a child?
$Q_{4,4,4,3}$: What effects are due to the person being elderly?