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A PROPOSAL OF ACTION TO INTRODUCE MODELLING IN SECONDARY CLASSROOM

Abstract. The official Spanish program points out the necessity to develop modelling process in real contexts. However, several studies suggest that secondary teachers have difficulties to implement modelling activities in their classes. In this work we present, throughout the description of an experience developed by one of the authors, a proposal of action to introduce modelling in secondary classroom. The final goal of this ongoing work is to design a teaching material that could be easily used by secondary teachers.

Keywords. Beliefs, modelling, classroom experiences, in service teachers.

Résumé. Une proposition d'action pour introduire la modélisation dans l'enseignement secondaire. Le programme officiel d'éducation espagnol insiste sur la nécessité de développer la pratique des processus de modélisation dans des contextes réels. En revanche, diverses études montrent que les professeurs du secondaire ont des difficultés à mener des activités de modélisation dans leurs classes. Dans ce travail on présente, en s'appuyant sur une expérience mise en œuvre dans une classe par un des auteurs, une proposition d'action pour introduire la modélisation dans l’enseignement secondaire. L’objectif final de l’expérience en cours est de développer un matériel didactique utile pour les professeurs du secondaire.

1. Introduction and theoretical framework

Modelling has dominated the human scientific activity since its origin and is deeply related to mathematics. In education, modelling is used to present the main results of science and technology to the students. Usually the process followed to construct the models and to arrive at this depurated knowledge is hidden from the students. However the process of the construction of a model contains very interesting mechanisms that can be used by teachers in order to make sense of different facts of Science. This is especially relevant in the case of mathematics that, immersed in the activity of modelling, appear to be closer to the role of a natural science than being a platonic construction.

During the last thirty years several authors have described mathematical modelling process -see for instance the works of Blum and Niss (1991), Lesh and Harel (2003) or Zbiek and Conner (2006)-. The main purpose of making use of a mathematical model is to tackle problems of the “real world” (or “extra mathematical world”). The modelling process can be divided in several steps: it begins with the interpretation and simplification of a problematic situation (real world problem). If appropriate, real data can be collected to provide more
information about the problematic situation. These data lead to a mathematical problem (mathematical model). Thus, mathematical methods are used to get a mathematical result of the problem. This mathematical result has to be interpreted in relation to the extra mathematical context of the real world problem, thus the solver must validate the model checking if the mathematical solution is reasonable or can be improved. Finally, the solver has to communicate and justify the solution. The entire process of solving a modelling task runs along a series of phases that are reflected in the so-called modelling cycle (Blum et al., 2002). However, this cycle is only a theoretical idealization, since it is not followed in a sequential and unidirectional way (Borromeo, 2006).

In the last years European education policies encourage the introduction of inquiry based learning (IBL) pedagogies in mathematics and sciences (Wake & Burkhardt, 2013). The IBL pedagogy is related to modelling, problem solving or even the use of technology in mathematics. A comparative analysis of French and Spanish secondary programs done by Cabassut and Ferrando (2013) showed that, in particular, modelling appears as a knowledge to be taught (in the sense of the Didactic Transposition, see Chevallard 1985, 1992). However, the changes in the curricula specification or even in the pedagogy that are encouraged in the initial teacher education seem not to be translated in the classroom (Dorier, 2010).

This distance between the official programs and the classroom practice can be explained by the teachers’ beliefs. As pointed by Schoenfeld (2010), teachers’ knowledge, goals and beliefs have strong effect on their classes because these three aspects are difficult to change. In the same direction Kaiser (2006) remarks that, for German teachers, “their beliefs concerning mathematics must be regarded as essential reasons for the low realisation of application and modelling in mathematics teaching”, moreover, she points that innovation required by the curricula can be interpreted by teachers in such a way they fit into their belief system, that is why only the parts of the program that are compatible with the existing beliefs are successfully implemented.

Several studies analyse the obstacles found by teachers when they try to introduce modelling in their teaching practice: “the fear of incompatibility of modelling tasks with the goal of mathematical exactness might be an obstacle for seeing productive learning opportunities in the tasks with higher modelling relevance” (Kuntze, 2011). Another important barrier for implement modelling in classroom relates to the resources; various researches have shown that most of the teachers have difficulties finding or creating didactic material that helps them to change their teaching practice. This can maybe explain the “conservationism” in education pointed by Dorier (2010) because teachers are used to routinize their classes. In consequence, the effective inclusion of modelling in the secondary school classroom entails defining a proposal to be applied in the present situation.
2. Motivation and objectives

The previous results imply that it is necessary to think about how to design a teaching material that can be useful to those teachers who don’t feel able to introduce modelling in their classrooms.

Modelling is a tool that is frequently used in science; researchers apply mathematical models to better understand the phenomena that they study. In some cases scientists don’t build a new model but they take a known model and they try to adapt it to their situation. We think that this way of work can be exported to the modelling in classroom. That is why our proposal of action includes what we have called “zero model”, this means a first model that, in our opinion, could help pupils (and, therefore, teachers) to work with modelling tasks.

Our purpose in this work is to describe an experience done by in-service teacher that is also one of the researchers. This experience is a part of an ongoing research whose objective is to design and to implement a teaching material that could be easily used by teachers without specific formation in IBL, in order to introduce modeling in their classroom. The main objective in this article is to identify the strong and the weak aspects of our proposal of action.

The next section is devoted to present the methodology of our research, there we will explain then we will describe the development of the experience in the second section through a quality-descriptive approach. The proposal of action was implement twice: first by one of the authors of this article (who is also a secondary teacher) and then, by two in-service teachers who were not familiar with modelling. The results of both experiences will be explained in the fourth section.

3. Methodology

This research has two differentiated parts: first we have designed a teaching material and then we have implemented it in order to investigate if this material can effectively be used by a teacher that is non-experienced in modelling.

3.1. Teaching material

The first part of the research corresponds to the design of the teaching material that will be implemented in a secondary school classroom. In the following we describe general aspects have been considered in the design process. In section four we will give particular details about one of the activities that was conducted during the experience.

For us, following the theoretical framework showed at the introduction, a modelling practice is an activity which start point is a real situation. The objective of a modelling practice is to introduce new mathematical concepts that emerge during the resolution process of the real problem.
In our experience, the modelling practice will be an activity conducted in small groups of two or three students. In fact, we think that this allows the collaboration between them to deal with the problem, that is, our experience fosters the cooperative work (E.F. Barkley and others 2007). This way of working gives pupils the option to discuss different points of view, compare their results and draw conclusions together, thereby facilitating the integration of students with different academic and cultural levels. The work groups are formed freely, depending on the own choice of students.

For the development of the activity, we have designed the following material:

- A teacher’s guide where it is described how to implement the specific modelling practice. The reason for giving this teacher’s guide is based on the conclusions stated in the first part of the paper. This teacher’s guide is similar to the material used by teachers for traditional lessons: we provide fundamental guidance to develop and to evaluate the practice.

- A student’s guide, where a “zero model” is described. This initial model is the starting point of the modelling practice. The purpose of this preliminary model is, from one hand, to help students to face the real problem and to give them the first mathematical tools that are necessaries to solve it. On the other hand, this “zero model” can be useful for those teachers that are not used to teach through modelling. More details about the role of this “zero model” will be explained later.

Modelling practices are organized throughout four different types of sessions: collective sessions, tutorial sessions, group sessions and oral presentations. The collective sessions are held during school time, every week during two months, and all students are present. In these sessions, the teacher answer general questions that arise in one group but can be of interest to the rest of the class -otherwise, the question will be answered only to this group-. One of the goals is to promote the collaboration and the debate between the different groups. The objective is also to create a weekly routine, that is, every week the students have to devote time to the modelling practice and also they have a common space with the teacher and their fellows to solve any problems that could arise. The whole sessions take place either in the classroom or in the computer room, where students can look for information and also ask questions that arise when they are using different computer programs.

Tutorial sessions take also place during school time, where every group works together with the teacher that acts only as a supervisor if help is required. At these sessions each group must show to the teacher the state of work and can ask about the doubts or about of which is the approach they could give to the problem. Thus,
if the group is stuck, the teacher can help them to consider possible ways to solve the problem.

Group sessions are held after school, that is, each group meets when they want and in some place chosen by them. The work developed by students in these sessions is difficult to be traced, that is why each group is provided at the beginning of the modelling practice with a diary. In this diary, the group members have to register how many times they have met, who was present in each session and some details about the subjects treated, the produced ideas, the debate inside the group, etc. The delivery of this diary is compulsory for all groups and will allow the teacher to guide the group during development of the practice and also to evaluate the final result.

The last part of the modelling practice corresponds to an oral presentation of the results. This session plays an important role in our modelling practices. The main goal is to give students the opportunity to show the work they have done. This session is carried out in the computer lab or in any classroom with media. The teacher and the rest of the students are present. Each group has about ten minutes to make their presentation. The rest of the students can ask questions at the end of the presentation and the members of the team have to answer them. The teacher acts as a chairman. The main goal of these sessions is to give the opportunity of students of arguing the conclusions and defending the results.

3.2. Research material

The second part of the work corresponds to the analysis of the implementation of the modelling practice. The teaching material described in previous subsection was implemented in two phases. First, during the year 2012, one of the researchers that is also a secondary school teacher, launch the proposal of action with a group of 9th grade students. Then, during the years 2013 and 2014 two different teachers were provided with the teaching material in order to use it also in two groups of 9th grade students. It is important to recall that this work is a part of an ongoing research. That is why, in this article, we will not completely develop the analysis of both experiences. We will mainly focus on some aspects that can help to obtain a preliminary evaluation of our proposal of action.

In order to describe and analyze the experiences the following material is available:

- Teacher’s diary which includes all the notes taken by the teacher during the collective and tutorial sessions. These notes can be related to the work of the students and also to the teaching material.

- Working group diary, where the students summarize the work done during the group sessions (held out of the school time).

- Video recordings of the oral presentation.
Final work presented in paper by the students.

In the following section we will describe the development of one of the modelling practices during the first phase of the experience, in order to give a more detailed explanation of our proposal of action.

4. Description of an experience

In this section, we introduce the experience based on a modelling practice carried out in a 9th grade class in the region of Comunidad Valenciana in Spain. As we claimed in the previous section, the objective of the modelling practice is to use modelling to introduce new mathematical concepts. In the present experience, the mathematical concept of function is on the basis of all the real problems presented to the students for modelling. Functions are present in all courses of compulsory secondary education. During grades 7 and 8, we start with the construction of tables of values and the interpretation of data and graphs, and in grades 9 and 10 students deal with the study of nonlinear functions and their interpretation in the real world.

Some aspects of the experience have been already explained at section 2.1. Here we begin with a general description of the experience and, then, we will give details of a particular modelling practice.

The first session of the experience is developed in classroom and is devoted to the formation of working groups and the presentation of four different modelling practices. All the modelling practices are related with the same mathematical concept but they differ in the real situation that leads to the real problem. During this first session each working group will decide which will be the modelling practice that they will work in.

Once every group has chosen a modelling practice, the teacher gives students the corresponding practice’s guide. This is a document where the work they have to develop is described, they can find the necessary material to start the job and it is included the development of a “zero model” or first stage model. With this material, they have to get familiar with the mathematical language, understand the information contained and try to deduce how it has been gathered. To summarize, in this first phase, they have to reproduce and interpret data and results contained in the guide. This will help them to understand a simple example. Then we ask them to create a new model, either applying the one showed in the practice’s guide, considering new data or observations or even creating another one totally different but inspired by the previous one.

The completion of the task concludes with a presentation of a document that is a report about the model and an oral presentation by the group members where they show to the rest of students their model and the achieved results. For the
development of these practices, students know how the activity is going to be evaluated since we give them a table where the items considered are listed.

We will illustrate this general description through one of the modelling practices that was proposed in the experience. We have chosen this practice because it was solved by one group of students during each phase of the experience.

4.1. Practical example

We show below the description of one of the modelling practices entitled "Footprints and mathematics" that is inspired by a task proposed in a PISA test (item 1 in 2009 test).

The starting point – the real situation or real problem – of the task is the question “how long a human step is?” and students have to work contents related to linear functions (concept of variable, linear dependency, parameters, etc). This is an open task in the sense that, from the starting point, students should consider other issues and learn how to solve them using the tools they judge necessary.

The task begins with some open questions to arouse curiosity of the students. Some of the questions are, for example: What is the length of your footstep? How many paces can you walk in a minute? How fast are you? These questions are often difficult to answer a priori. That is why, before beginning the modelling task, some ideas and specific easier questions are presented to them. The following table presents real data. It is important to note that the table shows different variables: weight, height, gender and number of steps but students have to decide which of them are relevant.

<table>
<thead>
<tr>
<th></th>
<th>Weight (kg)</th>
<th>Height (m)</th>
<th>Age (years)</th>
<th>Number of paces in 10 meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woman</td>
<td>65</td>
<td>1.67</td>
<td>36</td>
<td>13 14 13 13 13</td>
</tr>
<tr>
<td>Man</td>
<td>75.7</td>
<td>1.82</td>
<td>35</td>
<td>14 15 13 15 13</td>
</tr>
<tr>
<td>Child</td>
<td>26.6</td>
<td>1.28</td>
<td>6</td>
<td>16 17 15 15 16</td>
</tr>
</tbody>
</table>

Table 2: Data extracted from a real experience realized by one of the authors with her own family

Step by step the teacher explains in this “zero model” how to obtain the average length of the step of the woman whose data appear in the table. Then, the pupils have just to apply the same reasoning in order to obtain the lengths of the steps of the man and child. After this short introduction some questions arise: How far can walk women in 10 steps, and in 15 steps, and in 20 steps? At this time students have to distinguish two variables: one independent (the number of steps) and one dependent (distance travelled) and also one parameter: the length of one step. To
visualize the relationship between the variables and the parameter, the teacher asks them to build up a table of values and a graph. Once they have understood the calculations, the table and the graph, students are able to reproduce these calculations with the data provided in the case of the man and the child. After completing this section based on the table other questions arise. For example: “What kind of function have you obtained in each case?”, “Give a mathematical expression that relates the number of steps to the distance travelled.”, “Who is the fastest?” “Why?” At this point students just have to understand and use the “zero model”. They are not still modelling by themselves, but they are just doing what researchers use to do in science: take a simple model and try to fit it in order to understand a real situation.

Once they have understood the example, the process of developing the modelling task starts. At the beginning, in general, all groups try to repeat the above process using data they have collected by themselves. In order to arouse them to investigate other ways to find the length of a step, the address of several web sites are provided. Therefore they can observe that there is not only a unique valid method for this calculation, in fact they can try to create their own method. Once each group has obtained the step length of each group member, the teacher suggests them to use these data to calculate both distances and speeds. At this point it is important to compare both the estimate and the real measured distances so that they can see how small errors are amplified. It is interesting to notice that, at this moment the students begin to get away from “zero model”, they find other real problems related with the real situation and they will try to solve them using their own strategies.

The end of the work corresponds to some proposals that the workgroups have to choose, for example: to make a study about the pedometers (how they work, what variables are introduced, etc) or conduct research on the relationships that may exist between the length of the footstep and the length of the leg or arm. This section is clearly the most open, since the students have to decide what issues arise from the initial problem.

The task ends whenever each group make an oral presentation of the work to their classmates and teacher but the final mark is based both in daily work, contained in the student’s diary, in a written work and in the oral presentation.

5. Results
In this section we describe the result of the experience in two directions: from one hand we will show what the students did during the resolution of the modelling task previously described, and on the other hand we will comment how different teachers used our teaching material.
5.1. From student’s productions

We will present a qualitative description of the results based on the production of the students. We are particularly interested in showing what students are able to do from the “zero model” given at the beginning of the modelling practice.

When students faced the modelling practice “Footprints and mathematics” they did not have difficulties to understand the “zero model”. In fact, they were able to understand the variables given in the real data /weight, height, age, number of paces, the length of one step and the distance travelled, and they distinguished the differences between them. They realized that there is some relation of dependence between the variables.

At this stage, some students tried to obtain an estimate of the length of a foot step by average, counting the number of paces done walking 10m and 20m. This is very interesting because it show that pupils were able to relate two variables and, moreover, they tried to avoid the experimental error of measuring only one step by using an average.

Other students tried to go further searching for relation between basic human body proportions and the length of a step. This shows that, through the “zero model” they had the intuition of the relation between the height of a person and the length of the step. In order to find the relation they performed experimental measures and also they used Internet. Thus, they conclude that the pace’s length of a person corresponds to half the distance of the eyes to the soles of the feet. This shows how the “zero model” has served as basis to obtain a more accurate model of the situation.

Finally, a group of students tried to adapt the methodology learned in the work to other physical activities like running or swimming. The interest of this remains in the fact that students understand a real situation, obtain a mathematical model of the real situation and, then, they try to extrapolate this model to other similar situations. This is exactly the same that happens sometimes in scientific research and we consider that it is a valuable aspect of this experience.

We consider that it is also important to show the reactions of the students after finishing this task. In fact, the students that took part in this experience were not used to face this kind of activities, at least in mathematics classroom. The first part of the modelling practice was similar to the activities they are used to, but when they ask them to go further trying to building their particular model of the real situation, they were, in some cases, lost. At this point it is very important to consider the role of the teacher that implement the modelling practice: if the teacher is not able to guide the students working by themselves, they have serious difficulties to go further. This aspect will be developed in the next section. In the
following lines we show some examples of the comments of some of the pupils that attended to the experience:

- Ana: “I loved the task because I discovered interesting things and it taught us to apply mathematics to everyday things as simple as walking. The work was also too heavy because it required many hours of effort we suffered a lack of time.”

- Maria: “This is an interesting task since we learned things and we learned how to apply mathematics to simple act of walking or swimming. I had fun despite the conflicts that have occurred in this work both in the classroom and in the group. Despite this I liked.”

These are only two particular comments but, in general, we have observed that, from students’ point of view, it seem motivating to deal with these modelling practices.

5.2. From teacher’s point of view

Has we have explained in section 3.2., the modelling practice was implemented in two phases. First, one of the authors that is also secondary school teacher experimented the modelling practice in a 9th grade classroom. Then, in s second phase, two secondary school teachers used the same teaching material to work in their classes. This secondary phase has a great importance for this ongoing work because, in fact, our final objective is to design a teaching material that can be used by those teachers that are not really familiar with modelling.

We just make a preliminary analysis of the second phase of the experience here because, in order to get definitive conclusions, we need to increase the number of teachers that use our modelling practice. In any case, we have already observe some interesting things.

Both teachers that used our teaching material had any problem to implement the first part of the modelling practice, the one that corresponds to the “zero model”. In both cases, the teachers were able to make students aware of the different variables that appear in the real situation and the relations therein.

Nevertheless, one of the teachers was not able to manage the work of the students’ group in order to build their own models. This aspect is important because, from one hand, it shows the great importance of the first part of the practice, the “zero model” and, on the other hand, it manifest that we have to give a more accurate guide for help teachers to run the second part of the modelling practice.
Conclusions

Several studies show that even modelling is included as knowledge to be taught in official programs of secondary education (in particular in Spain), teachers find huge problems to introduce modelling in their classes. A way to bridge this gap between the programs and the real classroom practice is to propose materials that are easily implementable by teachers.

Our approach is different from other similar (as activities from LEMA project) in the sense that, besides proposing real life problems, it provides students previous knowledge, a “Zero Model” that they can adapt, extend and use to test their own hypotheses. Our methodology tries to mimic the activity of research teams in science as the natural way for the discovery and research, in the sense that the starting point is the knowledge achieved previously by other groups and researchers. This methodology, in our opinion, does not detract the student’s initiative and creativity, moreover, it is inclusive in two aspects: first, regarding student’s diversity (social, cultural, knowledge, etc.) and second, in the sense that it connects more effectively with classical methodologies (lectures, problem sessions, etc) which are customarily followed in the classroom.

From the results of the experiences done until the moment of writing this paper, we conclude that the teaching material based on modelling practice seems motivating for students and help them to learn abstract concepts such as the concept of function, variable or dependence. However we have also realize that, if we want to make a teaching material that is completely useful for teachers that are unaware of modelling, we must try to improve the instructions to deal with the second part of the modelling practice. In summary, zero model seems to be a useful tool to promote in students the modeling, especially in the aspect of being able to generate their own model and to extrapolate it from one real situation to another similar. This “zero model” results also useful for teachers in order to implement the activity in classroom but it results insufficient for helping them to deal with beyond this initial model itself.
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