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THE PRACTICE THEME IN MATHEMATICS EDUCATION : DEVELOPMENT OF A FRENCH-ENGLISH COLLABORATION ON THE ROLE OF THEORIES

Abstract. In this introductory article, we describe how the Special Issue came about, as a result of three years of collaboration. We show how this collaboration developed with a focus on the role of theory. We draw out themes across the individual papers, making use of the concept of 'boundary objects' that have allowed the comparison and enrichment of our different perspectives. We suggest that the work of bringing together researchers, from different traditions and on a sustained basis, is more important now than ever, in a context in which policy makers are increasingly looking at 'solutions' to problems in education from the other, seemingly more successful, countries (e.g. as measured by international comparison tests). We conclude with questions for further research, both for our particular collaborative group and more widely.

Keywords. Mathematics education, French and English approaches, theories, practices.

Introduction

This Special Issue is the result of a three-year collaboration between mathematics educators researching within French and English traditions (with French, British, Norwegian and Greek nationalities represented across the group). The initial reason we came together was to discuss and compare our theoretical approaches related to teachers' practices and teacher education. There was a sense that, across French and English traditions, there were similar areas of concern but with varied awareness of the detail of each other's work. The journey of arriving at the Special Issue has necessitated finding ways of working and questions or data allowing meaningful conversation. In this introductory paper our aim is to describe some of this journey in order to illustrate what has, and has not, occasioned useful collaboration. We use the theoretical notion of a boundary object (Star and Griesemer 1989) to allow us to analyze our work. We then offer some of the themes we observe across the papers within this Special Issue, leading us to implications for the future.

1. Journey to a Special Issue

The first meeting took place in January 2014, gathering European researchers having common interests but with different theoretical perspectives. We brought

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together ideas and issues relating to how theories are, or can be, used to investigate teaching practices and teaching development in mathematics. The aim was not just to present each participant's own work and ideas but to provide a forum in which ideas can be shared and new ones can emerge. Three work dimensions were at stake. The first was a focus on theories, enabling participants to bring to the table a range of theoretical perspectives and theoretical frameworks. The second was the substantive focus of our theoretical work; that is, teaching practices. The third was the interest in the use of theory to describe and understand how teaching develops and to question and inform the professional development of teachers.

Our meetings, together with the collaborative work in-between, first gave us opportunities to learn from each other. It also made possible groupings around specific interests and expectations. These groupings gave rise to common writings that are the articles in this Special Issue[1].

The duration of our collaboration turned out to be necessary, notably in order to shift from listening to each other to trying to find 'objects' that supported debate and the co-construction of comparative analyses. These objects might be qualified as boundary objects and, in some cases, technology enhanced boundary objects (Hoyles et al., 2010). Star & Grieseman (1989) define boundary objects as objects that allow communication across social groups and facilitate the resolution of different view points or conflicts in a creative manner. They "are objects which are both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites" (p. 393). Translating this quotation into the context of the work of the group of authors of this Special Issue, boundary objects can, for example, be analyzed across different research traditions and yet retain a sense of a common identity across these treatments.

One of the difficulties of communication across social groups or, in the case of this Special Issue, researchers coming from different traditions, is that objects used routinely within our respective traditions become naturalized (Bowker & Star, 1999), i.e., there are words or concepts used so routinely that their meaning becomes taken for granted. The further problem, within mathematics education, is that the same word may be used for different 'naturalized' ideas. A pertinent example within the group of authors of this Special Issue is the word 'theory' when talking about a mathematics lesson implementation. It took some time of working together to recognize the difference across traditions of what might be taken to be a

[1] We want to also acknowledge the role of colleagues, not represented in these pages, who joined meetings and attempted collaborations (e.g. around 'knowledge for teaching' and assessment) and found insufficient synergy to develop new insights. They each played an important part in discussions and in moving forward our collective thinking.

'theory'. A naturalized word for French researchers is that of the 'adaptation' of mathematics knowledge required in a lesson (which is subject to analysis, prior to teaching). While in the English tradition, this 'adaptation' is more likely, perhaps, to be conceptualized in terms of 'prior knowledge', possible 'misconceptions' and learning 'objectives'.

A boundary object is one that is not naturalized within any system or group but instead arises where social worlds, in our case research traditions, overlap. These objects are also not fixed over time and things may come into being as boundary objects between groups and then cease to function in such a way if they become naturalized in both settings (Star, 2010).

We can take here the example of 'teachers' professional learning'. This 'expression' often recurred in group discussions, about what the teacher learns after a lesson has been implemented. But no corresponding concepts seem to be theorized related to different approaches to learning or cognition. For instance, without being explicitly mentioned, there seems to be a largely constructivist conception of 'school' learning across the studies; perhaps this was a 'naturalized' assumption for most (not all) colleagues across English and French traditions.

At this point in our collaboration we are now aware of significant differences in our approaches to teaching, learning and research in the context of mathematics and mathematics education, as well as strong similarities in our concerns and overall interests. This Special Issue potentially offers readers a unique insight into a tradition of thinking with which they may be unfamiliar.

From an English perspective, the detail of the French analysis of teaching and learning moments comes through in every article. There is a recurrent pattern, more or less explicit, of the *a priori analysis* of mathematical tasks, i.e. the deep consideration of what mathematical knowledge needs to be brought to solve a particular task. Reading how this key idea is put into practice across the different articles allows insight into a French perspective and points to a recurring concern, within this perspective, about how such an a priori analysis could allow the teacher to negotiate the inevitable unpredictability of the classroom, when he/she attempts to bring students into contact with mathematical concepts.

In contrast, an influential idea from the past in the United Kingdom, perhaps from the 1960s and 1970s, has been the notion of students 'investigating' their own mathematics – a word that is, of course, interpreted in widely different ways. The current aims of one of the UK's subject associations (the Association of Teachers of Mathematics - ATM) include the statement : "The power to learn rests with the learner. Teaching has a subordinate role." (ATM, n.d.). We see here the idea that the teacher might, in some phases of teaching, be following the direction in which students choose to take a task, hence a detailed *a priori* analysis will sensitize

teachers to possibilities but perhaps not with the sense of predicting likely outcomes for a lesson. *Ways of working* on mathematics might be as much a focus for a teacher as the conceptual content. A question, from an English perspective, about *a priori* analysis might, therefore, focus on the extent to which there is always an intended path of learning for students and hence an inevitable 'gap' between teacher intention and student activity.

2. Themes and theories

This Special Issue is structured in a way that reflects the collaboration processes and their development. Article 2 (Jaworski, Lerman, Robert, Roditi, Bloch) is derived from the need felt at the beginning of the collaboration to understand the history of theory development in the French and English teacher education domain. It traces the development of theoretical perspectives in the English and French mathematics education research cultures from the 1960 and 70s to the present day.

Articles 3 to 6 present different ways in which groups of co-authors brought multiple perspectives into conversation with each other: juxtaposing, networking, analyzing the same data, meta-theorizing and, within each group, creating their own boundary objects. We elaborate briefly on the similarities and differences across these four Articles.

Articles 3 and 6 start from the same theory and show different interpretations and their use in different research.

Article 3 (Abboud, Goodchild, Jaworski, Potari, Robert, Rogalski) focuses on using Activity Theory to analyze classroom dialogue. Each of the French and English-speaking group uses their own data and analyzes it using their own understanding of Activity Theory, drawing out differences and allowing for reflections that would arise neither from English nor French traditions alone. The cultural context of researching seems to influence the interpretation of an understanding of Activity Theory, e.g. differences in view of the role of mathematical knowledge in Activity Theory applied to classroom practises.

The common interest in Article 6 (Mangiante-Orsola, Perrin-Glorian, Stromskag) is in the use of the Theory of Didactical Situations (TDS) to help structure classroom tasks. Common questions include how to design a-didactical situations (i.e., situations or tasks that hardly require the teacher's involvement, once set up) which, it is hoped, come close to 'guaranteeing' specific knowledge outcomes. The article leads to thinking about differences in: knowledge to teach; knowledge to learn; knowledge to act. How does TDS help thinking about these elements?

Articles 4 and 5 have the same focus but use different theories to address it. Article 4 (Abboud, Clark-Wilson, Jones, Rogalski) is based on the authors' mutual interest in investigating teachers' uses of, and practices with, digital technologies,

alongside the need to develop tools that could be used within teacher education programmes. They qualify their different approaches as ways of looking at the two different sides of the same coin, teachers' classroom practises with digital technology, from two different cultural perspectives. By working together, their aim is to see whether a knowledge of each side's facets leads to a deeper understanding of the coin as a whole.

Article 5 (Coles, Horoks, Chesnais) addresses the issue of making effective use of video for teacher development. The authors are interested in the role of the didactician-educator in working with video and, in particular, how theory is used in different ways by didactician-educators in the context of working with video. Through narrating and sharing different practices (with pre-service and in-service teachers), the authors develop ideas or questions to help tease out the role of theory in the work of a didactician-educator: What are the theories espoused by the didactician-educator? What theories are made explicit in any training session? What theories are intended to be used by the teachers (both for analysis and to inform their teaching)?

In the closing article, Jaworski and Robert offer a global overview of these last four papers, picking up some threads from Article 2, in which the authors presented key aspects of the English and French perspectives, to synthesize similarities, complementarities and differences. In part, then, the collaboration of authors has served to create new boundary objects by forcing naturalized words and assumptions to be questioned, both about the use of theory (Activity Theory or TDS) and, about practices with teachers and in classrooms (using video or using ICT).

Another theme more or less explicit in all four Articles (3-6) is that of 'contradictions' and 'tensions'. Article 3 includes an explicit focus on tensions and contradictions within mathematical representations, that can be put to use in a classroom. Article 4 theorizes cognitive pragmatic and temporal tensions in the use of ICT in a classroom, as well as the 'hiccups' that can occur to disrupt the smooth functioning of a lesson. The tensions in Article 5 occur across different levels of theory and, e.g. potential differences between an educator's espoused theory and what is enacted in their training sessions. In Article 6, it is clear that within TDS, the milieu enacted in the classroom is designed to provoke conflict and contradictions among students, leading to new knowledge. We suggest that such 'tensions' are another set of boundary objects across these articles, which have allowed authors to understand something of the detail of each other's practices as researchers and educators.

In fact, the inevitable moments of contradiction and tension that were generated during symposium meetings might themselves be viewed as the boundary objects, that provoked our communications, leading to the writing in this Special Issue.

Conclusion

We are intrigued that, at this moment (2018), policy makers in both France and England seem to be moving towards pedagogical approaches inspired by East Asian methods, seemingly taking less account of nearly 50 years of research in mathematics education in our two traditions. In England, there is a push to move towards 'mastery' teaching. The meaning of the term 'mastery' is, inevitably, contested but seems to capture a greater focus, than has been historically the case, on the details of conceptual development of students during a lesson or over the course of a term, year, school career. There are links here to the French research practice of *a priori* analysis of mathematics and attention to the careful sequencing of tasks, designed to provoke the development of mathematical knowledge, especially throughout the primary years. One important aspect of a priori analysis is the focus on consistent forms of representation at stake within a learning situation. For example, it is true that hands-on experiences at a primary level, using concrete objects, has an important role in an 'action situation' in the TDS (cf. Article 6 in this volume). However in this theory, 'action situations' are also conceived to engage students' initially available knowledge, with the aim that such knowledge will evolve, change or be rejected and replaced by targeted mathematical knowledge. Moreover, in the later 'validation situations' of TDS, it is this targeted mathematical knowledge which determines the types of validation, that themselves require language skills and representations, and not the representations which determine the mathematical knowledge.

In such a context, which we suspect is not unique to England and France, of countries looking to practices elsewhere, we argue that collaborations across researchers from different traditions are more important than ever. Such collaborations, merging and capitalizing on the results of research, are themselves a process of transforming and adapting, leading to proposals for initial or in-service teacher education as well as for the mathematics classroom.

To conclude, we quote here two thoughts from participants in this journey in the world of theories. One participant expressed their experience of what they gained from our collaboration as follows:

> 'I could explain what the other group means by 'Double Approach' (DA) , but the adherents of DA would say no, that's wrong; it is not what we mean. This has been our experience – we discuss, we hear, we interpret and we test the meanings we make by feeding back with our own words and the meaning we test is not that intended. Because we interpret from our own cultural immersion and feed back in the language of our cultural immersion, and the culture and language are not shared, we hear differently, we interpret

> differently and we express differently because we come to the discussion from different cultural positions. The discussion then produces a productive tension – tension because of the explicit disagreement, productive because it challenges the dispositions we have because of our immersion within a culture, of which we are unaware until the disagreement becomes evident.'

We see here again evidence for just how difficult communication is across research traditions and also how enriching it is, as we find the (boundary) objects and artifacts that allow us to collaborate. One other participant reflected:

> 'After many sessions discussing what we did with video, including doing some writing together, it was when we came to co-plan a session using video that there was a shift in our understanding of what each other does … because we could not find a video that we could both use! This dissonance felt highly productive and allowed access to some of the words each of us were using to describe what we did.'

Again, what we see here is the development of boundary objects, through a recognition that we had been making assumptions about meanings that were not shared - allowing access to deeper communication and the further development of thinking.

As we look to the future, we are committed to continuing and widening our collaboration; it is an open question as to what might now be productive areas of focus, what themes might bring together researchers and teachers and what new boundary objects might be needed. We welcome ideas, comments and communications from all readers.

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