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USE OF ACTIVITY THEORY TO MAKE SENSE OF MATHEMATICS TEACHING: A DIALOGUE BETWEEN PERSPECTIVES

Abstract. This paper examines the interactions between teachers' decisions, discourses and acts, and the intended students' learning. The focus is theoretical and methodological as it attempts to exemplify theoretical perspectives in studying mathematics teaching in its complexity. It takes into account, together or separately, the overall setting: sociocultural and institutional and the epistemological point of view on mathematics and its teaching in class. For some of the authors, the study of teacher activity in relation to students' mathematical activity, and affective and social needs has been the focus of their research for many years, using different theoretical constructs and empirical data. As for the others, their research in the same area was focused more on the presumed cognitive needs, in relation to the practices and the mathematics at stake. The article reveals that Activity Theory has been used differently by the two traditions (English and French) as a framework for analyzing and interpreting the relations and interactions between teacher and students' mathematical activity in research studies of the authors. This article exemplifies these different ways of using AT and discusses issues the perspectives raise for interpretation and analysis.

Keywords. Teacher activity, student activity, cognitive aspects, social aspects, affective needs

Résumé. Deux perspectives pour l'utilisation de la théorie de l'activité dans l'étude de l'enseignement des mathématiques. Ce texte est centré sur l'étude des relations entre les activités des enseignants et celles des élèves, les premières étant décrites en matière de décisions, de discours et d'actions. Il s'agit d'adopter un point de vue théorique et méthodologique, en lien avec les perspectives adoptées pour ces analyses complexes ; cela fait intervenir, sans qu'il y ait exclusion d'un des aspects, l'ensemble des déterminants socioculturels et institutionnels, les déroulements en classe et le point de vue épistémologique. Une partie des auteurs fait notamment intervenir dans l'étude des pratiques enseignantes les besoins affectifs et sociaux, l'autre insiste davantage sur les besoins cognitifs présumés et les mathématiques en jeu. Tous les auteurs se réclament de la théorie de l'activité comme cadre théorique pour analyser et interpréter les relations et interactions entre l'activité enseignante et les activités mathématiques des élèves. Nous illustrons chaque point de vue par un exemple en discutant des questions qui se posent à l'autre point de vue.

Mots-clés. Activité de l'enseignant, activité de l'élève, aspects cognitifs, aspects socioculturels

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Introduction

In this paper we, English and French researchers, present briefly the different ways that Activity Theory (AT) has been used in our research and exemplify them through the analysis of two data extracts. The extracts have been chosen to be illustrative of our approaches and provide opportunities for contrasting them. Indeed, our collaboration has demonstrated that the contrasting approaches in using AT results in the need for different qualities and characteristics of data generated for our empirical purposes. Thus, it became clear to us very early in our collaboration that we could not easily share data that had been generated specifically for either the English or French perspectives. The first extract comes from a group tutorial session at a university in the United Kingdom where first-year students work on tasks of finding partial derivatives of a function. The second extract comes from a high school classroom in France where the focus is a lesson (i.e. moment of teacher exposition) on the sign of an inequality of the second degree. Even if the situations are quite different (work on tasks for United Kingdom, a lesson for France) the teachers' goal in both extracts is for the students to make sense of the underlying mathematical ideas, while the students' goal is less visible to the researchers. In both cases the teachers are more or less guided by what students say and do, and act to enable students to achieve the teachers' goals for the students. Research questions are closely related to the theoretical perspective adopted and consequently the English and French groups are concerned to address different research questions. The English group is concerned with the nature of teaching in the tutorial and how this is linked to student mathematical meanings. The French group is concerned with the distance between what students do and/or know and the teacher's goals for the students during a lesson; and how students' responses to the teacher influence the actions and mediations of the teacher in trying to reduce this distance.

For the English group, the analysis is framed by Leont'ev's work on consciousness as the basis of personal knowing and establishing notions of Activity Theory (AT), which is built on Vygotsky's psychological interpretation of Marxist dialectical materialism. This articulation of AT is manifest in categories of actions and goals, division of labour, inner contradictions and mediating tools. These categories are used in the analysis of the first extract looking for relations and tensions (that emerge from the activity's inner contradictions)[1] between the teacher and students' activity and how these tensions were resolved.

Constructs from Vygotsky's work and the French Didactics, such as ZPD (Zone of Proximal Development) and the 'Double Approach' are used for the analysis of the second extract. The approach entails mathematical analysis of 'relief'; that is, the specificities on the learned notion intersecting with curricular and students' difficulties. The approach is also concerned with the dynamics between conceptual and applied aspects and corresponding occasions of proximities (between a student's present and intended knowledge or conceptualization). The French approach thus shows, again the process of bringing closer, the teacher's actions and the students' expectations and needs (Bridoux, Grenier-Boley, Hache, & Robert, 2016).

The two perspectives do not have the same starting point or the same focus when investigating class activity (teacher and students). The French perspective is firstly concerned with students' activities in order to detect what characterizes and what differentiates teachers' practices, according to the adopted hypothesis on students' learning (conceptualizing). Whereas the English perspective begins from the teacher's activity and the mathematics she is dealing with, to study what occurs in the class in terms of students' activity. More globally, a critical analysis of the different ways of using AT will be developed to look for the similarities and complementarities of the different perspectives and on how they contribute to our learning about the complex relation between mathematics teaching and learning. Such a reflection will contribute to possible new ways of theoretical networking.

1. Activity Theory

In Sections 1.1 and 1.2 below we present, first, the English (1.1) and then the French (1.2) perspectives on Activity Theory. We explain briefly in each case the main theoretical constructs that underpin our use of AT to characterize the activity of mathematics teaching-learning. The English and the French perspectives relate approximately to different levels of the general frame of AT as grounded in the work of Vygotsky and Leont'ev, and later developed in some contrasting ways in the English perspective and in the French tradition. Considering Leont'ev's three layers of activity (Activity-Motive; Actions-Goals; Operations-Conditions: Leont'ev, 1978, 1981), the French approach is centred on the actions and operational layers whereas the English one also gives consideration to the motives and goals of activity. Moreover, the French analysis focuses on the teacher/student

[1] Beside this theoretically based concept of tension, the everyday notion of tension is used in this chapter (and in chapter 4) as denoting the idea of divergences between intentions or goals.

relationship within the classroom related to mathematical objects within teaching and learning issues. Tensions are seen to be situated in the gap between what could be initiated from students' mathematical actions and the mathematical aim of the teacher. From the English point of view, the tensions are considered to emerge from contradictions arising within a larger activity system including institutions; they deal with the specific goals of each teaching-learning event within the system and relate to the mathematical objects at stake. Even if the two perspectives refer to the same theoretical source, Vygotsky, they follow different paths.

The English perspective presents itself in line with the evolution of AT as developed at a general theoretical level in section 1.1 below. Nevertheless, this perspective focuses on classroom interactions, seeking to analyze interactions in terms of the more general concepts of AT. The French perspective is presented in section 1.2, it starts from Vygotsky's theory but focuses on his developments on conceptualization and the key notion of ZPD (Vygostky 1986, chapter 6). This theoretical input leads to question precisely the tasks presented to students and their intended mathematical activity. These contextualized tasks and their implementation in class may be considered as tools mediating the teaching-learning activity. The ZPD starting point is developed in an epistemological way that analyses how the teacher makes use (or not) of possible proximities between students' previous knowledge and the mathematical content at stake. These proximities could be considered as didactical devices that the teacher uses to bridge the gap mentioned above.

1.1. Activity theory from an English perspective

Our analysis of mathematical discourse in a university tutorial seeks to explore and explain the exposition and appropriation of mathematical meaning by tutors and students respectively. In doing so, we take a socio-cultural approach, that is cultural-historical activity theory, which emphasizes consciousness as the basis of sense making and hence personal mathematical meaning. Roth and Radford (2011), in their articulation of AT, explain that 'consciousness' in activity is theorized as "the relation of a person to the world" (p. 18). They argue, based on their interpretation of the work of Leont'ev, that consciousness is the basis of personal knowledge, rather the cognitive and constructivist positions that invert the relation by positing knowledge (schema) as the basis of consciousness: "consciousness, …, is not characterized by comprehension, not by the knowledge of the significance of the subject matter, but by the personal sense that the subject matter obtains for the child," (Leontyev, 1982, p. 279, in Roth & Radford 2011, pp. 17 &18). Consciousness emerges within 'activity', which is the sole, indivisible unit of analysis, or in Leont'ev's terms, "the non-additive, molar unit of life" (Leont'ev, 1981, p. 46). Our purpose here is to theorize the university mathematics tutorial within terms of AT; for a deeper discussion about the principles of AT the reader is referred to more comprehensive expositions such Roth and Radford (2011), and Leont'ev (1982).

Activity takes place over time and is pursued to achieve an object that results in an outcome or product in the material world, and its realization is its motive in the psychological consciousness, "an activity's object is its real motive" (Leont'ev, 1981, p. 59). In the present case, we see 'activity' as university education in mathematics, as manifested in the tutorial. The motive here is the education of students in mathematics with the object of their enculturation into the mathematical worlds developed historically and seen through the eyes of the research mathematicians who teach them in the university.

Different actors within the activity may seek different, not necessarily contradictory outcomes, for example: engineers and scientists equipped with the necessary skills to contribute effectively to national and societal development; a deep understanding of mathematics; sufficient mathematical knowledge to achieve a degree result that secures employment or admission to further study. AT is rooted in Vygotsky's psychological interpretation of Marxist dialectical materialism, and points to the division of labour, inner contradictions and tools that mediate between subject and object of activity. These characteristics of activity are fundamental to understanding the educational transactions that occur within a mathematics tutorial.

Mathematical ideas are presented in various representations such as graphs, equations, symbols, and expressions, which are the tools that mediate mathematical meaning. However, embedded in these tools are contradictions rooted in mathematics as well as didactical transactions (teaching actions and operations). The tutor may use mathematical representations to lead the students to a deep understanding of the mathematical ideas. Students may also be expected to communicate their consciousness of the ideas using these same mediating representations. However, the representations are not the mathematical ideas that the tutor wants the students to understand, they need to understand and be able to use the representation at a surface level, they also need to become aware of the mathematical concepts represented at a deep level.

In her attempt to address the inner contradiction of the representation the tutor may use a didactical tool, 'inquiry'. She will pose questions about the representations and mathematics and try to provoke curiosity and inspire the students to ask their own questions. However, the division of labour in the tutorial in which the tutor is cast as the expert who teaches and the students are novices who do the learning creates the context for the contradictions of inquiry. The teacher's questions may be intended to cause students to reflect on their own mathematical meanings, to articulate them, and by bringing them into the open allow them to be examined and give the students an opportunity to review and revise them. The student, however, may confuse the tutor's question as an attempt to evaluate. The student may also be reluctant to share naïve meanings because of the reaction of his/her peers in the tutorial.

In Vygotsky's analysis of activity, the division of labour results in contradictory perceptions of the material product in a material transaction; for the producer, the product has an exchange value, it is worth what the producer can get in exchange for it. For the buyer, the product has a use value. The common category in the contradictory meanings of the material product is the notion of value, the transaction occurs because for both producer and buyer the product has 'value'. At this point the contradictions of the transaction in the mathematics tutorial - between teacher as a producer of mathematical contents and students as buyers - may not share a common category, especially if the tutor and students have different goals. For the tutor, the goal may be that the students develop a deep understanding of mathematics. The tutor is experienced, informed and in possession of her own deep understanding of mathematics. On the other hand, the students' goal may be 'instrumental' in acquiring that consciousness of the representations and relationships that will enable them to be successful in an examination. The different goals imply a different consciousness of mathematics. Is it possible to consider a common category 'value' of mathematical competence if the meanings of competence held by tutors and students are so different?

Returning briefly to the theoretical grounds of AT, it is possible the above discussion could convey a notion of activity being a structure of distinct elements – actions that combine into events, operations such as asking questions, and tools such as mathematical representations. Such a notion would be incorrect. The activity exists as actions and the actions can only be understood within the context of the activity, as activity endures over time the actions take place in time. As the activity is established on achieving some object, the actions are directed to achieving goals. Actions are achieved through carrying out operations which are subject to constraints and mediating categories embedded with the activity – the rules, division of labour, tools and acting people's consciousness. Each of these categories can be understood only in the context of the indivisible unit of analysis – activity, and the analysis of activity entails examination of each of these categories and the dialectical relations that exist between them.

1.2. Activity theory from a French perspective

Hypotheses and theoretical approaches

Framing our research in an AT perspective leads us to firstly study class episodes when students are solving mathematics exercises. Indeed, from the perspective we adopt, this kind of students' activity is what determines, for a great deal, their learning (Vandebrouck 2012; Abboud-Blanchard et al. 2017). The analysis considers both the tasks provided and their implementation in lessons. The latter are studied with reference to the expected students' activities deduced from task analyses and from the observed management of the teacher. The context (programmes, mathematical notions involved, and particularities of the school, the class and students) is also taken into account. But between the planned activities and what the students really do, there exist many differences and diversities. We do not have access to the actual individual activities of the students (of each student) but we try to apprehend their possible activities which are associated with the teacher choices in terms of statements, exercises, discourse (mathematics or not), students' work format and management (including what comes from the students themselves). Moreover, these choices are conditioned both by the desire to make students learn and by constraints related to the teaching approach (see Double Approach Robert & Rogalski 2005). These constraints may lead to choices based on, for example, curricula, class heterogeneity, time constraints, and working in a peaceful atmosphere, choices that are not directly related to students' learning.

Studying episodes of exercise solving, enabled us to have a growing knowledge of both students' and teachers' activities, accomplished within these class moments (Robert 2012; Abboud-Blanchard & Robert 2013; Chappet-Pariès, Robert & Rogalski 2013; Chappet-Pariès, Pilorge & Robert 2017). However, there remain other crucial moments in class learning, those of the exposition (specifically, lectures and lessons) or moments of 'telling' when the teacher is directly presenting some mathematical content. The methodological challenge is to study these moments while simultaneously taking account of the mathematics at stake, teaching and learning, and the broad context within which the lesson occurs. The student activities are often invisible and therefore inaccessible. The usual a priori task analysis does not apply here, and yet it is indeed the organized set of lessons and exercises that contribute, in a long-term process, to the intended conceptualization (learning), which is our actual object of study. Indeed the decontextualization and the general formulation (institutionalization) of the elements of mathematics involved (e.g. definitions, theorems, properties, formulas, methods etc.) are indispensable to this process.

We look to AT to conceive and provide the tools to analyze these moments. We draw inspiration from Vygotsky's theories (1986) and especially from the ZPD model to propose a hypothesis that shapes our study. In order to analyze these class moments, we focus on the teacher's discourse that presents the knowledge to be learned, tracking his/her role as a mediator between the specific (contextualized) and the general, and between the old and the new. Indeed, we admit that the challenge entailed in the exposure of new knowledge is to get students to appropriate and use connections between words, formulas and general statements and particular contextualized mathematical tasks proposed to them. We must consider what may have happened before and what would happen afterwards in the classroom; the connections may emerge at first provisional and partial, during and after the course. In other words, the more the teacher succeeds in bringing together the general elements at stake with what students already know or have already done, including contextualization, the more the conceptualization (learning) aimed at could progress. That could be done by means of comments, of making explicit connections with existing or future knowledge, by explanations of the use of some statements, noting what is invariant or related to historical references, and so on. We call 'meta' all the elements of the teacher's discourse about mathematics and about mathematical work (see Robert & Robinet 1996; Robert & Tenaud 1988). The 'effectiveness' of the lessons, conceived as elements of a long process, then depends on the opportunities, involving the chosen tasks, and the quality of all teacher's mediations.

In order to carry out such a study, it is necessary to provide tools to analyze the content of the lessons (supplementing the tools for analyzing 'exercise-type' tasks) and their implementation.

Methods

The data we collect is mostly a video recorded by the teacher herself with a static camera at the back of the classroom, its transcription and, if possible, a teacher's account of what has preceded the lesson and of the context of the class.

First, we study what we call the *relief* (or landscape) of the mathematical notion to be taught, combining therefore a threefold analysis of this notion: epistemological, curricular, and the already known difficulties that students experience when meeting this notion. This enables us to estimate the distance between what students already potentially know and the new concepts to be introduced, and to reflect on this introduction. It is also important to understand if and how the difficulties the students may experience are taken into account within the lesson. These analyses are subsequently used, on the one hand, to characterize each specific lesson to be studied, with its precise environment, and to have an idea of the possible alternatives. As for the transcriptions, a first examination makes it possible to specify the modalities of the implementation, the moments of exchanges, listening, copying the dialogue or even the repetitions, which makes it possible in particular to track down what comes from the students (answers or questions).

Once these two stages of the analysis are completed, we try to detect the teacher's choices related to the approaches taken in the lesson. We pay particular attention to what can be more or less qualified as attempts of alignment that the teacher operates between what has been done in class and what he/she wants to introduce. We distinguish in particular the connections between general and particular and those which are made at the same level of generality. These are what we call the discursive proximities that we will detail in the following. Notice here that it is the researcher who interprets, on the basis of the relief she/he has already established that there may or may not be such alignment or need for alignment; the search for what is implicit is thus valuable in this respect. What is at stake here may concern: the level of generality of non-contextualized statements, rigour and vocabulary, written versus oral properties, and anything that can illuminate the functioning of the presented knowledge, in particular its status (accepted, demonstrated or presented without comments), and its usefulness for future applications or for consistency throughout the course.

The proximities are hence elements of the teacher’s discourse that could influence the students’ understanding according to their existing knowledge and their activities, which are in progress. This occurs in the operationalization of the mathematics class within the presumed ZPD. Three types of proximity are to be distinguished in the way the teacher organizes the movements between the general knowledge and its contextualized uses: we call ascending proximities those comments that make explicit the transition from a particular case to a general theorem or property; descending proximities is the other way round; horizontal proximities, however, consist of repeating or illustrating the same idea in another way.

The study of the transcription in a more detailed way gives access to what happens during the lesson. More precisely, we can distinguish between the proximities introduced by the teacher from the outset and the proximities arising from students' answers to the teacher's questions or resulting from students' spontaneous questions. Thus the researcher can have a fairly accurate view of all the proximities, of what motivates them and of what remains implicit in the studied lesson.

This enriches the comparison between different lessons and classes, from the same teacher or between teachers. The developments of these theoretical tools enable us to target the gap between what students do and/or know and the teacher's actions and mediations. The theoretical tools also facilitate the study of the moments of knowledge exposure through the development of analyses in terms of discursive proximities. Moreover they enable us to appreciate opportunities for possible or even missed proximities between what is general and stated by the teacher and what the students already know or do.

2. Examples illustrating the perspectives

2.1. Discussion of analysis with regard to theory in English perspective (cf. 1.1)

The analysis is illustrated through an episode from university mathematics teaching within a tutorial setting with first-year mathematics students in England. The students are expected to attend lectures in calculus and linear algebra and work every week on problem sheets that their lecturers have set. In the tutorial the tutor (third author) works with students (one hour per week) on material related to the lectures, often taking questions from the problem sheets that according to her would reveal key concepts in mathematics and might cause difficulties for her students. The episode comes from the tutorial in Week 6 of Semester 2. Four students and the tutor are present in this tutorial. The tutor has chosen to work with the students on questions from the problem sheet set by the lecturer of the calculus course involving differentiation of functions of two variables. The students work together on the following question:

Question: The three graphs of Figure 1 show a function f and its partial derivatives f_x and f_y Which is which and why?

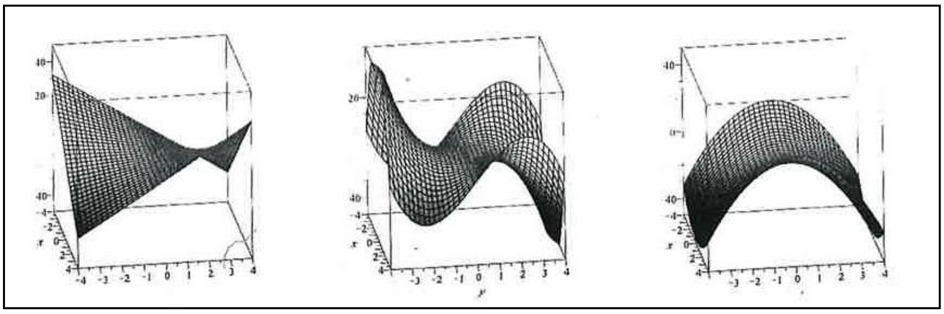

Figure 1: Extract of the problem sheet

A transcript from the first 6 minutes of the tutorial is presented in Appendix A. In this we see a dialogue between a tutor and 4 students in a university small-group tutorial focusing on distinctions between partial derivatives of a function represented graphically. In the analysis, the tutor's knowledge of the mathematics at stake is accepted. The tutor also has knowledge of the students, which developed through engagement with them through the previous semester, and this knowledge guides her engagement through the tutorial. The tutor's goal is that students will develop a deep understanding of the mathematics through engaging in a critical manner with the graphical representations, transformations, mathematical language and expressions that are used in the question (Fig. 1), the students' presumed prior knowledge and the content of the course they are currently studying.

The main research question that is addressed here concerns the nature of teaching in the tutorials (including the characteristics of teaching – what the teacher does, her *actions* and associated *goals*, how mathematics is addressed, what tools she uses to engage students and encourage their understanding) and how this is linked to students' mathematical meanings. Initially, we analyze the episode line by line using a grounded approach to see the actions and goals of the tutor and the students' responses, and to start to interpret them. The approach, which we have used throughout our research over many years, takes the data as a point of departure, and begins with a process of data reduction out of which the main themes emerge and are subsequently categorized using open coding. Essentially the approach does not apply any theoretically rooted categories until after the initial open coding. Then we use constructs discussed in Section 1.1 in the context of the Activity of university mathematics teaching, and of tutoring in particular, and its motive, student learning and understanding of the mathematical concepts; the tools that are used to achieve goals; the emerging contradictions between the tutor's goals and the students' responses. All three stages, data reduction, open coding and application of theoretical constructs, were undertaken independently by three analysts (authors 2, 3 & 4), before meeting to agree the interpretation set out below.

A grounded analysis of the episode – a summary

The following figure presents the first 6 turns of tutorial transcript, the complete 6 minutes transcript is reproduced in Appendix A.

1. T: [Tutor and 2 students are present] I thought we'd have a look at Q3 first. I've selected all of these questions for a purpose, because each one of them highlights what I would call key concepts. [She refers to question 3 as presented above. Two more students enter the room – tutor greets them and repeats her words above]
2. T: So, first of all what are these things fx and fy? Alun. What is, what do you mean, if you write fx and fy?
3. S: (Alun) dee-f-dee-x
4. T: And how would you write it?
5. [He indicates with his hand the partial derivative symbol, ∂]
6. Yes partial df/dx and similarly fy is partial df/dy. When you say df/dx so you want to be clear, we would say here partial df/dx and partial df/dy [She writes on the board ∂f/∂x and ∂f/∂y]

Figure 2: First 6 turns of tutorial transcript

Turn by turn scrutiny of the transcript reveals the following characteristics of the dialogue:

- Tutor (T) states her goals for her approach in the tutorial (turn 1).

- Tutor questions to students (turns 2, 4, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33)[2].
- Student responses to tutor questions (turns 3, 5, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32).
- Tutor explanation/clarification of concepts (turns 6, 7, 33).
- Tutor focusing on 'meaning' explicitly (turns 2, 15, 33) or implicitly ('why' questions: turns 13, 21, 25).
- Student responses that (start to) reveal meaning (turns 5, 12, 14, 16, 18, 26, 32).

These details reveal an alternating pattern of tutor questions and student responses; some of the latter not revealing student thinking about the concepts. Those that do reveal some potential insights for the tutor become the focus of further tutor questions.

14.	E: … because it is got the, er, the slants of the first one, and the…
15.	T: so you're seeing a relationship between the one of the middle and the other two. What do you mean by the slants?
16.	E: er, I don't know, just the, the gradient there.
17.	T: if you're right and the function is middle one, erm, before we go any further, Alun, do you think the function is the middle one or would you say one of the others?
18.	S: (Alun) … it looks like the more complex
19.	T: aah..“It looks like the more complex”. So would you expect the function graph look more complex than its two …?

Figure 3: Tutorial, turns 14 to 19

The tutor tries to prompt meaningful student articulations, but this is only partially successful. Student use of language “slants” (turn 14), “gradient” (turn 16) and “complex” (turn 18) suggest meaning to the tutor who probes and prompts with further questions (turns 16-19).

The teaching approach here can be interpreted as a questioning approach that prompts students and probes their meanings (Jaworski & Didis, 2014). It tried to include students by addressing them singly, by name, and as a group. Further interpretation suggests students either do not know the answers to the questions posed, or are not able to articulate their understandings. The tutor mainly avoids providing her own answers to questions posed, seeking rather to draw out the

[2] We have included all instances of each type of turn here to emphasize the frequency with which these occur within a 6-minute episode (see Appendix A).

students' own articulation of meaning. However, in the university culture in which they all participate, it is unusual for students to be asked to articulate their mathematical thinking, so perhaps not surprising if they show inability or unwillingness to do this.

As the tutor is also one of the researchers, she provides information about her goals in the tutorial teaching in general and in the episode in particular. Although the tutor is not 'teaching' the calculus course, she has a responsibility to help the students make meaning of the mathematics. So, her questions, as well as seeking out what the students know (what they can express in words), also have the purpose to assist conceptualization. She works according to a belief that a focus on 'meaning', with direct questions encouraging students to express meaning, will bring meaning into the public domain in the social setting.

Activity - actions and goals - tools

Activity here is the university mathematics teaching and in particular the tutoring. The object of the activity is student enculturation into the professional community of mathematicians; the motive of the Activity is the development of scholarly knowledge of mathematics. The participants/subjects of this activity are the tutor and the students and, following the above analysis, the episode comprises actions directed towards their reciprocal goals of communicating and appropriating understanding of selected key mathematical concepts related to partial derivatives of functions of two variables and their associated graphs.

We perceive an enculturative process to involve development of mathematical meanings as the objective of mathematical activity (rather than perhaps the limited goals of procedural functioning). In this particular episode the tutor's goals are to get students to:

- express what they 'see', their images, their connections, their symbolic awareness, their thinking;
- get used to talking about the mathematical concepts, to express ideas in words;
- link to formal mathematics ideas;
- listen to each other and build on what another person expresses;
- feel comfortable about not knowing, but to recognize that working together can enable more than they could do alone.

These are goals for the students, but the tutor also has goals for herself:

- to phrase questions in ways to which students can respond;
- to listen to the students and discern meaning from what they say;

- to maintain a focus on the mathematics that is important, without telling, guiding, funnelling in ways that will foster a surface recognition without deeper meaning.

In order to gain access to students' meanings and develop further their mathematical meanings she needs some tools. One tool is the question of the problem sheet, part of which is the three graphs as an iconic representation as well as symbols and terms that are used. Her questioning approach is another tool.

The tutor's actions relate to these goals. Her main action is to ask questions, and the different kinds of questions relate to different goals. For example, the prompting and probing questions seek to engage the students in thinking about the mathematical concepts and taking part in the tutorial dialogue. The 'why' questions seek to discern students' mathematics meanings through their articulation of reasons for their answers to her questions. Her use of the lecturer's problem sheet both aligns with the expectations of the university system in mathematics and provides a source of opportunity for students to address the mathematical concepts of the calculus module. The limited offering of her own explanations and exposition is intended to elicit explanations from students rather than providing them herself.

The goals of the students are not made explicit in the episode, and we do not have the relevant data to talk explicitly about them. Nevertheless, as the tutor has observed from tutoring these students for a whole semester and from her other tutoring experiences, the students show more satisfaction when they see how to apply certain procedures and find the solutions of the problems given than to develop deep understanding of the key concepts that the tutor wants them to achieve. Their main goal in participating in the tutorial is to be successful in the class examinations. As we discuss below, these different goals arise from the inner contradictions of the activity and they cause tensions that the tutor needs to handle. Tensions emerging from inner contradictions are also related to the way that the students handle the representations (tools) that the tutor offers to them. Also, the students bring informal tools such as informal language and images in their attempt to make sense of the key concepts that the tutor wants them to understand.

Contradictions, tensions and convergences

There are emerging tensions for the tutor that are of pedagogical and didactical nature. She is familiar with these students and is aware of the factors which influence their participation; the demands on them from their other courses; their difficulties in understanding mathematics, expressing formally and engaging analytically. Her approach has to take into account the wider context. There is no point in manifesting expectations that the students have no chance in meeting. She might be drawn into her own explanations and expositions which the students will not understand any more than they understand the lectures they have attended. Nevertheless, she has to be aware of the key mathematical ideas, and keep the focus on these ideas. Keeping a focus may be in tension with fostering students' own articulations of meaning. Maybe there are other strategies (tools) she could employ, and she does so at other times in this tutorial and in other tutorials. In contrast with her own values in seeking conceptual meaning, the tutor has to be careful to ensure that students see some value in the time spent in the tutorial, otherwise they might not attend on future occasions. Thus, she has to ensure there is some outcome of positive value perceived by the students, even if it is not clearly in line with her main goals. So, for example, students value tutor actions that enable them to answer questions in a test or examination, and they might prefer to gain procedural awareness of how to address a mathematical question without caring for the deeper understanding. So, sometimes it is necessary for the tutor to focus on procedural competency such as how to differentiate a two variable function with respect to one variable. This is something they have done in a previous tutorial.

Another contradiction related to the representations concerns whether the students understand the key concepts that the tutor wants them to articulate or their attention is on the representation itself. The tutor's focus is on symbols – meaning appears to be emphasized with the word 'partial', and later by the idea of imaging (not imagining) planes parallel to x-z and x-y. There is further focus on interpreting graphical representations – features in terms of 'dominant' shape, zeros, stationary points (and types). Distinguishing between the graphical representations of f and its partial derivatives appears to rest on a notion of complexity. It is not possible to grasp or present the key (ideal, generalizable) concepts, it is only the representations that the tutor can express, point to, inspect, etc. Thus the tutor is confronted with the fundamental contradiction in teaching mathematics. What does she do to bring the key concepts to students' consciousness?

We have seen in the transcript above some of what the tutor does and how the students respond. It is hard to judge the outcomes from these actions in terms of the expressed goals. To what extent are students enculturated in mathematics according to the motive of activity? Activity is, of course, ongoing and not limited by the beginning or end of a tutorial. The wider story must deal with actions and goals beyond this tutorial.

2.2. Discussion of analysis with regard to theory in French perspective (cf. 1.2)

We will illustrate the approach we developed for studying moments from mathematics lessons using one example. In such moments the teacher presents to the students general and somehow formal mathematical knowledge. The access to students' and class's activities is more limited than in exercise sessions. Students listen (or not) to the teacher, copy onto their note sheets what is written on the blackboard, perhaps take notes, and think about what the teacher is telling: but these activities escape the classroom video-recording.

Context and content of the recorded lesson

The lesson we use to illustrate is with a 10th-grade class. The declared aim of the teacher is to bring students to use a sign table in order to solve an inequality composed by the product of two factors.

An introductory phase, that was not recorded, took place around the solving of the following problem: A firm wants to make mouse pads consisting of a square image of side 10 cm framed by a strip of colour of constant width. The width of the coloured strip is x cm. For economic reasons, the area of the large square thus formed must not exceed 225 cm². Determine the possible widths of the coloured strip.[3]

The teacher gives the following account of this phase. First, students were given a few minutes to reflect on the problem and then a discussion ensued. A resolution scheme is then sketched, followed by setting the inequality: $4x^2 + 40x < 125$. After having made a value table, students drew the curve of the function $x \rightarrow 4x^2 + 40x$ and tried to solve graphically the inequality by drawing the straight line: $y = 125$. A question of the teacher guides the students' activity: show that the inequality is equivalent to: $(2x - 5)(2x + 25) < 0$. Students are encouraged to solve the case where the product is equal to zero, and then to apply the rule of signs. The teacher draws a sign table by recalling the lesson on the previous chapter about the sign of an affine function and checks that the solution is consistent with the graphic resolution.

In the lesson that follows this activity, first the teacher presents the graphical resolution of general inequalities such as $f(x) > k$ and $f(x) < g(x)$ by the means of curves. Then he writes on the blackboard the next title: algebraic resolution of inequalities. In the first paragraph he presents two tables showing the sign of $ax+b$ according to the sign of a. It is only then that the recorded episode starts; a full transcription is provided in Appendix B.

The teacher recalls, with the students' participation, the rule of signs with numbers, seen in the introductory phase. Then he presents a more general proposition on the rule of signs with a product of two factors A and B (numbers or algebraic expressions) and provides a summary table that the students copy. Then follows the

[3] No student was using a geometrical solution: maximum area is 225 cm^2, hence maximum length is 15 cm, so maximum x is 2,5 cm. It can be inferred that it is an effect of the didactical contract (at this school level) that the approach has to be algebraic.

statement of a method, deduced from this generalized rule of signs, to determine the sign of an "algebraic expression product", which is introduced through an example: find the sign of $(2x + 1)(x - 4)$. After a short discussion about the methods (to develop, to factorize) proposed by the students, which the teacher refutes or comments upon, he returns to the proposal to make a table of signs. He makes precise the nature of the factors involved (as “affine functions”). He recalls, through a series of quick questions to the students, that if the slope is 2, positive, the corresponding affine function is increasing. He then prepares an empty table of signs that the students copy. It is then completed by both teacher and students. After a question from a student who did not understand, everything is repeated once more.

The aim of the analyzed episode is to learn how to design and use a sign table in order to determine the sign of a product of expressions of degree 1 (ax+b), the so-called “rule of signs”.

The relief of the mathematical content at stake

Students are supposed to recall what they have learned about linear functions and especially what was done previously for their sign, leading to the algebraic resolution of an inequality as $ax+b > 0$ with the corresponding table.

Students are expected to be able to recognize and use the rule of signs for numbers. In fact, for some students it is probably not “available” knowledge, particularly if numbers are not given as numerical values (such as +3, -7) but expressed as a, b, without any explicit sign. They are also expected to move fluently between three registers: “the number a is positive”, “a is greater than zero”, “$a \geq 0$”, and to associate the signs “+” and “-” as indicating a position with regards to zero (for instance, in +2, the sign + indicate a positive number, greater than zero, such as $+2 > 0$).

In the curriculum and in the textbooks, the “rule of signs” for numbers has already been seen in earlier years (an item of “old” knowledge). As concerning linear functions, they are first introduced at grade 9; their study is developed for 10th-grade students, not only relating to the algebraic formula and the graphical representation, but also introducing the value of the zero of the function as the value where the signs change. A specific aim is to introduce the construction of the sign table for a product of linear functions.

Indeed, what may be difficult here is the difference between the direct algebraic study of an inequality composed of a single linear function and the algebraic study of an inequality composed of a product of such functions, which, moreover, may not be directly visible in the given algebraic form. The impossibility to solve the second type directly, leading to a detour with the use of an extension of the sign rules, remains difficult for a long time. Furthermore the link between the graphical resolution and the algebraic one is not obvious, as the first one does not involve the product of linear functions.

The lesson in progress

The teacher introduces the session with a rule expressed for the "product of positive and negative things" (A and B). Then he points out that "A and B are numbers or algebraic expressions", and announces a "method to determine the sign of an algebraic product of factors [...] something times something, a product". It is done by extending the rule of signs and is based on what is known for the sign of affine functions. The presentation is developed for the specific case of a product of first order simple expressions ($2x + 1$) and ($x - 4$). The teacher quickly draws the table on the blackboard for students to copy it, comments on the number of lines and announces that "the method is to have one line for each factor: a line for $2x + 1$ and another line for $x - 4$", without commenting on the role of the first line (x values) and of the last one (signs of the product), until a student questions the teacher's announcement, "I bring down the zeros on the bottom of the table". Finally, he recapitulates the whole process by answering a student who apparently did not understand anything.

Proximities

We track in the teacher's discourse elements which we presumed were oriented toward making links between previous knowledge and the mathematical content presently at stake. We name them "discursive proximities".

The proximities directly expressed by the teacher were of various types:

- an ascending proximity concerns the rule of signs, when he expresses the similarity between the (yet known) rule for numbers and the new rule for expressions;
- the teacher then announces that the method will be deduced from this rule: another ascending proximity;
- for the table of signs, there is a descending proximity between what students know about the sign of an affine function (recalled just before); a horizontal one - at a general level - is involved when he says, "the method is to put one line for each factor";
- the importance of the values of zeros is commented with a descending proximity, "in order to use the table of signs for the affine function, as we had done (just before)";
- the same proximity is used for fulfilling the line of signs for each factor
- another descending proximity is present for the sign of the product "we apply the rule of signs".

The proximities linked to students' utterances:

- In the case of answers, two descending proximities appear when the teacher interacts with students for studying the sign of each factor and for completing the line of x values with the two zeros in the appropriate order;
- We identify a local horizontal proximity triggered by students' questions, when the teacher relates the term "product expression" to the product known as "something times something, a product"; a descending proximity when the teacher explains why the question "for what value is there a change of sign" was changed into "for what value is it zero?";
- Responding to a student who did not understand, the teacher resumes his explanation, adding several proximities. Two descending proximities are involved in the application of the sign of affine functions previously learned "we wrote just now, and we wrote in the lesson on affine functions, that the sign is ..." and in the generalization of the rule "if I get 15 cases after the zero, I put as many "plus" as there are cases". Two local horizontal proximities were also present: the teacher explicits that before x of x - 4 there is "1" as a coefficient ; he explains that the rule of signs is used along columns as for the null values of a product ("if I take a thing that is zero times another thing that is not zero what does it give?");
- Elsewhere, we observe a refused descending proximity, when a student proposes to use the general form of solution -b/a for the zero of $2x + 4$.

In fact we see that the students have a real influence on the teacher's explanation during the lesson, giving rise to the teacher's descending or local horizontal proximities. However, we notice that there is no questioning related to the students' previous work (possibly giving rise to ascending or general horizontal proximity). This reveals somehow the limits of what could be initiated by the students' questioning. Actually there are notions, properties and notations that remain implicit in the lesson, as it is presented below, what would perhaps involve horizontal general proximity.

***Implicits*[4]**

There is a diversity of implicit use of notions, properties or notations, some of them being evoked later on in the lesson.

- A first implicit concerns the A and B expressions: the reason why it is possible to use the previous knowledge about signs lies in the fact that they are

[4] We use here the substantive "Implicits", it is a neologism – the plural is built on the model of "deficits".

supposed to be expressions with the same variable (x), a notion that does not belong to the students' curriculum.

- A second kind of implicit is related to the use of mathematical registers: “>0”, “greater than zero”, sign +, “positive”; and the notation of the line of x from $-\infty$ to $+\infty$.
- The explanation of the relation between variation of an affine function and graphical representation enabling the visualization of the change of sign is not given. Perhaps it is supposed available as affine functions were introduced in the previous grade and worked on before the session, and also in the first part of the lesson?
- How to use the so-called “method” for solving inequality problems remains implicit, even if the session is just followed (or even preceded) by a specific example.

Some of these implicits could be considered as “missed proximities”, mainly horizontal ones. The appropriate moment for such proximities remains an open question.

If we come back to our “relief” on the algebraic resolution of such inequalities, we may suppose that what some students could miss is more the idea of the necessity of a detour by the study of the appropriate product by the extended sign rule than the technical way (sign table) to do it, which was more developed here by the teacher. It could have given rise to some horizontal general proximity, linked with an appropriate task. We suppose that an appropriate assessment may be used to check this kind of hypothesis.

Conclusion

The theoretical and methodological perspectives presented above and the examples used to illustrate their use shed light on different ways to analyze and interpret the interactions between teacher and students’ mathematical activity. Even though the two perspectives follow different routes, with a shared origin (Vygotsky’s theory), some similarities seem to appear and some questions remain, particularly about the notions of contradiction and tension (without, however, considering the same level of generality).

Through their example, the English group reveals emerging contradictions for the tutor that are of pedagogical and didactical nature. In particular we ask:

1. What do we learn from articulating these contradictions? Why is this of value more generally?
2. What insights does the revealing of contradictions provide with regard to teaching for students’ understanding of mathematics?

The French example pointed out different types of proximities in the relationship between teachers' goals and students' real activity. We can hence add a question:

3. What, if any, are the kinds of proximities that are less likely initiated by students' interventions, and therefore need to be initiated by the teachers?

In relation to Question 1, the fact that there are contradictions in teaching is not new or surprising. We have seen the revealing and naming of them in previous research, particularly at school levels (Brousseau, 1984; Jaworski, 1994; Mason 1988). An example is the so-called "Didactic Tension" deriving from Brousseau's (1984) Topaze Effect as observed by Mason (1988) and used by Jaworski in her analyses of teaching (1994). In this paper we reveal contradictions in *university tutorial* teaching, which is relatively new, and the use of Activity Theory aids this process. Activity Theory, as we have shown above, in its various manifestations, draws attention to contradictions (and resulting tensions) in educational practice (e.g. Roth and Radford 2011).

In Section 2.1 above we see contradictions between teacher actions goals and the responses of students and between teacher actions goals and teachers' interpretation of the meanings behind these responses. We also see inner contradictions in the ways in which mathematics is presented and perceived (that representations are not the mathematics they represent, but that students may see the representation as the mathematics). In starting to generalize, we suggest that the declaring of contradictions is of value more widely, firstly, as the research and teaching community acknowledges the importance of being aware of contradictions and secondly recognizes them in other research or in their own practice. Thus we start to form a classification or knowledge bank relating to contradictions in teaching at a range of levels and opening the debate on how teaching can address such contradictions, whether they are inevitable or whether they can be avoided. In doing so we start to form a theory of teaching in which contradictions are seen as unavoidable, but in which we seek teaching actions that can better address teaching goals.

It seems worth exemplifying these generalities in terms of the examples above. The tutor has certain goals for her work with her students. These include the desire that they develop deep understandings of concepts such as partial differentiation. Her associated actions include the selection of suitable mathematics tasks chosen to reveal the desired concepts; orally delivered questions designed to prompt and probe students' understanding; grasping small clues in their minimal responses (slants; gradient …) in order to judge their understanding and offer further prompts, etc. Whether students develop understandings, deep or otherwise, from this activity is not visible. Hence the teacher cannot decide whether her actions have achieved her goals, or whether some other actions might be needed.

In the tutor's example we can stress the consciousness of the practitioner reflecting on teaching decisions and actions in relation to expressed goals. Here we address Question 2 above. There is considerable debate in university teaching as to whether traditional lecturing achieves learning outcomes that a university desires. The above discussion on actions, goals and associated contradictions offers an important contribution to this debate. From the conceptualization of theory on contradictions and their importance in educational development we envisage a dialogue between practitioners in which the teaching community becomes more aware of the vicissitudes of practice and potentially more critical in their design of teaching to achieve desired learning of mathematics by student cohorts.

In the French analysis we also reveal tensions in teaching lessons, which is relatively new (previous research has been centred on relationship between mathematical tasks and students' activity in classroom exercise sessions). The use of Activity Theory, in relationship with Vygotsky's theorization about conceptualization, aids and supports the analysis. The proposed theorization of proximities would be a model of teachers' mediation aiming at provoking evolution in students' knowledge, from recently acquired mathematical notions ('old' ones) to new ones. It proposes a more fine-grained model than the Vygotskian dyad: spontaneous and scientific concepts. The tensions occur between what is expected or planned by the teacher,[5] what appears to be possible or not according to the students' answers or own questions, what has to be improvised by the teacher to articulate the specific and the general levels of mathematical objects at stake, or between 'old' knowledge and new, through discursive proximities.

Two elements particularly emerge from the analyzed teaching situation. First, there remain some implicit issues in the teacher's discourse, at moments when 'old' knowledge might be mobilized or reinforced; these mainly concern the general level of mathematical objects or activity.

Second - and this is some answer to the third question - students do not appear to make spontaneous connections between existing and new knowledge, or their mathematical actions, and it is up to the teacher to explicitly introduce these connections. In these moments of mathematics lessons, the teacher's activity is neither triggered nor completed by students' initiatives - questions or comments. Establishing proximities appear then, crucially, as the teacher's initiative in articulating knowledge for the (expected) students' benefit.

[5] The data used for presenting the notion of proximities are not analyzed from the point of view of the teacher's expectations and planning, we are referring to our general approach in the studies of teachers' practices.

To conclude, we can say that looking for relations and complementarities between the English and the French approaches to analyzing mathematics teaching through the different uses of AT, led us to recognize connections between proximities and contradictions (and resulting tensions). The notion of proximity is a construct that indicates how the teacher tries to bridge the gap between students' existing mathematical knowledge and the mathematical content that the teacher wishes to communicate, tracked through the teacher's discourse elements. Recognizing different types of proximities, tells us about how the teacher attempts, in different ways, to overcome these tensions and build bridges. The proximities allow us to scrutinize the teacher's actions in relation to his/her attempt to introduce students to new mathematical meanings, taking into account the students' mathematical activity. On the other hand, with the constructs of contradictions, actions, goals and their relationships, the English approach allows us to recognize tensions that are also beyond the classroom interaction and play an important role in the interaction itself and its outcome. Through the different constructs of AT, the analysis contributes to our understanding of the complexity of mathematics teaching. Focusing on critical moments in classroom interaction we identify mathematical, didactical, and institutional factors coming into play that inform teachers' decisions and actions and, as a result offer learning opportunities for the students.

Appendix A

Transcription of an extract of a recorded tutorial in first-year university mathematics

A transcript follows from 6 minutes of classroom dialogue in a university small-group tutorial focusing on partial derivatives.

1. T: [Tutor and 2 students are present] I thought we'd have a look at Q3 first. I've selected all of these questions for a purpose, because each one of them highlights what I would call key concepts. [She refers to question 3 as presented above. Two more students enter the room – tutor greets them and repeats her words above]
2. T: So, first of all, what are these things fx and fy? Alun. What is, what do you mean, if you write fx and fy?
3. S: (Alun) dee-f-dee-x
4. T: And how would you write it?
5. [He indicates with his hand the partial derivative symbol, ∂]
6. Yes partial df/dx and similarly fy is partial df/dy. When you say df/dx so you want to be clear, we would say here partial df/dx and partial df/dy [She writes on the board $\partial f/\partial x$ and $\partial f/\partial y$]
7. So in the question then, we have three graphs; one of them is a function f and the other two are the partial derivatives df/dx and df/dy. Now, which is which?
8. [silence]
9. T: Anybody have a stab at that? What do you say Brian? [He pulls a face and people laugh]
10. [Response unclear]
11. T: No? OK, how about you Erik?
12. E: … not really sure but I guess that, er f will be the middle one.
13. T: OK, why do you think that?
14. E: … because it is got the, er, the slants of the first one, and the…
15. T: so you're seeing a relationship between the one of the middle and the other two. What do you mean by the slants?
16. E: er, I don't know, just the, the gradient there.
17. T: if you're right and the function is middle one, erm, before we go any further, Alun, do you think the function is the middle one or would you say one of the others?
18. S: (Alun) … it looks like the more complex
19. T: aah…"It looks like the more complex". So would you expect the function graph look more complex than its two …?
20. S: I would.

21. T: you would. Why?
22. S: [pause] I don't know.
23. T: do you agree with him, Carol?
24. S: yeah (Carol)
25. T: can you say why?
26. S: erm because it has in this x and y, functions of both x and y.
27. T: well, don't they all?
28. S: more functions, …
29. T: more functions?
30. S: er, I don't know!
31. T: Come on we're getting there. Brian?
32. S: Well, I guess when you differentiate, you're almost simplifying it to your next .[inaudible]
33. T: OK, so if what we have got is, in some sense a polynomial, then when we differentiate a polynomial we get a lower degree, so is that what you meant by 'simplifying'? So is everybody agreed then that the middle one is the function?

    OK. It is!! It is.

    So look to the one on the right, Erik, and tell me how the one on the right fits with what you see in the middle. Is that going to be the partial derivative fx or is it going to be the partial derivative fy?
34. [The dialogue continues in the same style for 4 more minutes]

Appendix B

Transcription of an extract of a recorded course in a 10th-grade class

(Statements of students are in italic – comments of the observer are in italic placed in brackets)

Transcription of an extract of a recorded course in a 10th-grade class

(Statements of students are in italic – Comments of the observer are in italic placed in brackets)

<table>
<tr><th>Time starting from the beginning of the recording</th><th>What the teacher says</th><th>What the teacher writes on the blackboard</th></tr>
<tr><td>4’38

4’46

4’56
Silence 10’’</td><td>So do you remember what we have said earlier about the product of positive and negative things (students give some answers)

We have told negative times negative is positive, negative times positive is negative, positive times positive is positive.
So it is what we call the rule of signs
So we made a small proposal, placed in brackets you can write : rule of signs,
not the animals [swans, in French "cygnes" same pronunciation as “signes”] - sign rule
(she erases the blackboard)
and we will draw a table
(She draws on the blackboard without saying anything)</td><td>Sign of a product

<table><tr><td>Sign of A</td><td></td><td></td><td></td><td></td></tr><tr><td>Sign of B</td><td></td><td></td><td></td><td></td></tr><tr><td>Sign of A.B</td><td></td><td></td><td></td><td></td></tr></table></td></tr>
<tr><td>5’40

6’15

Silence 25’’
6’47</td><td>So A and B are numbers, or algebraic expressions and the question is about the sign of their product
So you said that if the two are positive the product is positive. If the first one is negative and the second positive it gives negative, if I reverse it, again it gives negative, and if I take two negatives it gives positive. That is what you have just said to me.
A student’s question (inaudible)
Yes. I said if A is positive, B positive, A times B is positive. Minus times plus is minus, plus times minus is minus, and minus</td><td><table><tr><td>Sign of A</td><td>+</td><td>−</td><td>+</td><td>−</td></tr><tr><td>Sign of B</td><td>+</td><td>+</td><td>−</td><td>−</td></tr><tr><td>Sign of A.B</td><td>+</td><td>−</td><td>+</td><td>−</td></tr></table></td></tr>
</table>

Silence 12'' 7'10 7'25 Silence 15'' 7'52	times minus is plus. (*the teacher is silent, the students copy*) And we will deduce a method to determine the sign of an algebraic product of factors. *Student : But Madam, it is normal, in fact it is simple* Yes, I don't disagree. You have known that for a long time, but there are things you do know from a long time, yet you do not know how to use them. So a method, method to determine the sign of an algebraic product of factors (*she dictates*) To do the method we will take a very specific example. We'll take an expression and we will do the algebraic study. (*She repeats*) Method to determine the sign of an algebraic product of factors. Product that is to say something times something, a product. So what example I could give.	
8'30 Silence 15'' 8'33 8'49 9'10 9'23 9'40	Let us find the sign of (2x + 1) times (x -4). I'll wait until everyone has finished writing. *Student: That's in the lessons' part?* It is always in the method, the method, we apply it on an example. *Student: we multiply the factors together?* Chaima, ah, certainly not! *Student : we factorize then* What do you want to factorize? *(Inaudible answer)* We'll make a sign table Actually we use what you know about signs. So the first part: it is a function ..? *Student : affine* Affine. The slope here is equal to… ? (*Student : 2*) 2, is positive so the expression is first negative, then positive, an increasing function. This one is also affine. The slope is equal to…? *(Student :1)* 1, positive, so it is also negative, then positive. At which value the sign changes? *Student : it's –b/a* Yes, there is no need; it is also possible to solve the equation. When does it give zero? *Student: at 4* When x is 4, and this one?	Let us find the sign of : $(2x+1)(x-4)$ We solve $2x+1=0$ $\Leftrightarrow 2x=-1$

<table>
<tr>
<td>Silence 17’’</td>
<td>Student: When x is equal to 0.5.
When x is minus 0.5. We write it
First we solve 2x + 1 = 0 (she writes it) and x -4 = 0 (she writes and leaves a blank). The first one gives 2x = -1;
x = -1/2; -0.5; and that one is much easier, it gives x = 4 so we get both values .</td>
<td>$\Leftrightarrow \quad x = -\frac{1}{2}$

et $x - 4 = 0$
$\Leftrightarrow \quad x = 4$</td>
</tr>
<tr>
<td>10’35

10’51

11’
Silence 25’’
11’52
Silence 12’’
12’16
Silence 17’’</td>
<td>These two values are important.
Student: what is the use of the zero then?
Should first find for what values it is equal to zero, in order to use the sign table of the affine function like we already did.
Student: Why affine ?
Each piece, each factor, we look when it is equal to zero in order to determine the sign and so we deduce the sign table.
(she draws the table and leaves some time to copy)

Then, it is a table that will have 4 lines; however a nice big table. If you still have two lines at the bottom of your page, I do not know if it will hold.
Then the method is to have one line for each factor: a line for 2x + 1 and one line for x -4
(she leaves some time for students to copy) .</td>
<td><table><tr><td>x</td><td>$-\infty$ $+\infty$</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td>
</tr>
<tr>
<td>12’50

Silence 5’’</td>
<td>Then we write the signs. To fill the lines with signs, we begin by putting the two values; which one first ?
Student : -1/2
Why?
Student : Négative
Especially because it is smaller than the other one. I write the smallest first. -1/2 then 4, with lines below.
Please be careful, you must try to put it just underneath, otherwise the table become unreadable.</td>
<td><table><tr><td>x</td><td>$-\infty$</td><td>$-\frac{1}{2}$</td><td>4</td><td>$+\infty$</td></tr><tr><td>2x+1</td><td></td><td></td><td></td><td></td></tr><tr><td>x-4</td><td></td><td></td><td></td><td></td></tr><tr><td>(..)(..)</td><td></td><td></td><td></td><td></td></tr></table></td>
</tr>
<tr>
<td>13’33</td>
<td>Let us write the signs. We start with 2x + 1, 2x + 1 is equal to zero at which value?
Student: At -1/2
At -1/2, so at -1/2 in the line of 2x + 1 I put a zero. Only at -1/2 eh since it is equal to zero only at -1/2 . Then I fill in with the signs. It's minus, plus, since the slope is positive so here it gives minus, minus, plus.
I repeat, if we take 2, 2 is positive therefore according to the sign table we had earlier on the affine functions it gives minus, plus, plus.</td>
<td><table><tr><td>x</td><td>$-\infty$</td><td>$-\frac{1}{2}$</td><td>4</td><td>$+\infty$</td></tr><tr><td>2x+1</td><td>–</td><td>0</td><td>+</td><td>+</td></tr><tr><td>x-4</td><td></td><td></td><td></td><td></td></tr><tr><td>(..)(..)</td><td></td><td></td><td></td><td></td></tr></table></td>
</tr>
</table>

<table>
<tr>
<td>14’17

14’30

14’45

15’09</td>
<td>Now the second one
Student: we put zero at 4.
We put zero at 4, it becomes null at 4 and ...
Student: here it is going to be minus, minus, plus.
Minus, minus, plus (she is writing) and the slope is 1. Student: We do the sign rule.
And in the third line we put the product, in fact we apply the sign rule.
And the last thing, I bring down the zeros on the bottom of the table.
Student: why do we do it?
Because if this one is equal to zero at -1/2, if I make the product by the other, the product of the two is ..., if this one is equal to zero at -1/2 if I multiply it by (x-4) it will still give ... (Student: zero)
And here it is the same for 4, so it is zero at the two values we had found.
Is it okay? No, why? What's wrong? What piece did you not understand? (Student: Inaudible)
Then how do we write the plus? Why did you say it's minus, plus?
Student (another one): You put plus when it is greater than zero, minus when it is smaller.</td>
<td>
<table>
<tr><td>x</td><td>$-\infty$</td><td>$-\frac{1}{2}$</td><td>4</td><td>$+\infty$</td></tr>
<tr><td>$2x+1$</td><td>–</td><td>0</td><td>+</td><td>+</td></tr>
<tr><td>x-4</td><td>–</td><td>–</td><td>0</td><td>+</td></tr>
<tr><td>(..)(..)</td><td></td><td></td><td></td><td></td></tr>
</table>

We apply the sign rule
<table>
<tr><td>x</td><td>$-\infty$</td><td>$-\frac{1}{2}$</td><td>4</td><td>$+\infty$</td></tr>
<tr><td>$2x+1$</td><td>–</td><td>0</td><td>+</td><td>+</td></tr>
<tr><td>x-4</td><td>–</td><td>–</td><td>0</td><td>+</td></tr>
<tr><td>(..)(..)</td><td>+</td><td>0</td><td>– 0</td><td>+</td></tr>
</table>
</td>
</tr>
<tr>
<td>15’41

16’

16’11

16’50</td>
<td>The slope here is 2. 2 is positive. We wrote a while ago, and also in the course on affine functions, that the sign is minus than plus. That means minus before zero, after zero it is plus. If I have 15 boxes after the zero, I get 15 plus, I put as many plus as there are boxes after the zero. Basically it's minus, then plus. This one now. Again the slope,1, is positive, so it is again minus then plus. Minus before the zero, plus after the zero.
As for the last line, we applied…what have we applied in the last line? (Student: the sign rule) (she writes it) . We apply the rule of signs in columns: minus times minus is plus, plus times minus is minus, plus times plus is plus. An the zeros, we bring down them because if one of the factors is equal to zero then the product is also null. If I consider something equal to zero and something not, then it gives…? (Student: zero)
Student : we must systematically bring down the zeros to the bottom of the table.
As for the product, yes!
Student: and if there are more factors?
I can put 15. There are many more values and</td>
<td>She adds one before x in the expression x-4</td>
</tr>
</table>

17'35	the table is much larger. If I take a product with three factors, then I'll have a third value here and I'll have a third line here but the rule of signs will work the same, that is if I have plus, minus, minus, minus times plus is minus, these two together give minus, when we multiply by minus it gives plus. The rule of signs functions for more than two factors	
19'26	End of the recording	

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