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ANALYZING TEACHERS' CLASSROOM EXPERIENCES OF TEACHING WITH DYNAMIC GEOMETRY ENVIRONMENTS: COMPARING AND CONTRASTING TWO APPROACHES

Abstract. The use of digital technologies in mathematics classroom continues to increase. Yet even when well-planned, such use is not unproblematic; indeed, uncertainties are inherent. In this article, we use analyses of teachers' activity in two classrooms, a French one and an English one, when technology in general, and dynamic geometry software in particular, is used. We present two different theoretical frames and show how, in spite of differences related to the context, the object, and the methodological backgrounds, the outcomes in terms of the analysis of teachers' practices turn out to be close. These outcomes provide insights into the complexities of technology integration within mathematics lessons and teachers' decision making both in the moment, and over time.

Keywords. Technologies, geometry, teachers, activity, hiccups, tensions

Résumé. Analyser l'activité instrumentée de l'enseignant en classe dans un environnement de géométrie dynamique : différences et similitudes de deux approches. L'utilisation des technologies numériques en classe de mathématiques continue à se développer. Cependant, cette utilisation reste complexe et demeure régie par des incertitudes lors des mises en place avec les élèves même quand les séances sont bien préparées en amont. Cet article présente les analyses de l'activité de deux enseignants, un français et un anglais, lors de séances intégrant des logiciels de géométrie dynamique. Nous présentons deux cadres théoriques et montrons que malgré les différences liées au contexte, aux notions mathématiques en jeu et à nos choix méthodologiques, les résultats en termes d'analyses des pratiques enseignantes sont très proches. Ces résultats fournissent un éclairage sur la complexité de l'intégration des technologies dans les séances de mathématiques et les décisions que les enseignants sont amenés à prendre in situ et sur le long terme.

Mots-clés. Technologies, géométrie, enseignants, activité, tensions, perturbations

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Introduction

The genesis of this article lay in the authors' mutual interest in each other's work as researchers, work that involved a close look at teachers' uses of, and practices with, digital technologies alongside the more pragmatic need to develop tools that could be used within teacher education programmes. In some sense, our methods

ANNALES de DIDACTIQUE et de SCIENCES COGNITIVES, Special Issue English-French
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look at two sides of the same coin, the teachers' classroom practices with digital technology, from our two different cultural perspectives. By working together, our aim is to see whether a knowledge of each side's facets leads to a deeper understanding of the coin as a whole.

The integration of technology into classroom work is known to be a complex process for teachers (Hoyles and Lagrange 2005; Clark-Wilson, Robutti and Sinclair 2014). While some studies have explored the nature of these complexities (Abboud-Blanchard 2013; Clark-Wilson and Hoyles 2017), a key to supporting the development of classroom practice is the availability of methodological tools and framing ideas that enable teachers to both understand the complexities and develop practices as a result.

Teachers of secondary mathematics in England and France have incorporated dynamic geometry software environments (DGE) into their teaching practices, including use by students to support them to engage with, and make sense of, geometric ideas (Laborde, 2001; Ruthven, Hennessy and Deaney 2008). Teachers, often following curriculum guidance, design DGE-based geometrical tasks where students are working in an investigative mode involving conjecturing and generalizing. Teachers support students throughout this investigation in different ways; for example, by introducing new mathematical objects or showing (or possibly proving) a geometric property. However, by opening the mathematics to student exploration, teachers encounter the pedagogic challenge of how to manage multiple student responses to tasks within the technology.

In this paper, we report findings from our analysis of teachers' activity in two classroom video sequences, one from a French classroom and another from an English classroom, using two different theoretical frames.

The first frame (the French context) is informed both by the *Double Approach* (Robert and Rogalski 2005) extended to technology environments (Abboud-Blanchard 2013) and the *Instrumental Approach* (Rabardel 2002). It considers teachers' use of technology as managing 'open' dynamic environments (something that increases uncertainties for the teacher in the classroom) and can be used to analyse teachers' activity in terms of *tensions* and *disturbances* in the planned cognitive route of the class (Abboud and Rogalski 2017).

The second frame (the English context), which is underpinned by Verillon and Rabardel's theory of instrumented activity within technology-mediated environments (1995), introduces the theoretical construct of the *hiccup*[1] to capture

[1] In English, the word hiccup (or hoquet in French) has the additional meaning: a small problem or difficulty that does not last very long.

the epistemological rupture experienced by a teacher as he/she develops professional knowledge in practice, stimulated by the students' use of mathematical technologies (Clark-Wilson 2010a, 2010b).

The two frames are both affiliated with the theory of instrumented activity (Rabardel 2002), as is detailed in each of the two examples. We explore how the difference between them has a methodological implication as it concerns differences in the relationship between the researchers and teachers they are investigating. In particular, we use the different foci for these two research studies as each 'enters' the mathematics classroom to try to understand aspects of the teachers' (and students') knowledge at stake when technology in general, and dynamic geometry software (DGE) in particular, is used. While the context, the research objectives, and the theoretical and methodological backgrounds differ, the outcomes (in terms of the teachers' practices) could turn out to be close. This raises the prospect of whether the two theoretical perspectives can be connected in some way.

1. Characterizing teachers' classroom experiences with dynamic geometry technology: An example from France

1.1. Theoretical approach

In this example, the theoretical approach, informed by the *Double Approach*, considers the teacher as managing an 'open dynamic environment' (Rogalski 2005). Indeed, the use of technology adds a 'pragmatic' dimension (Abboud-Blanchard 2014) emphasizing the 'open' character of the classroom environment. On the one hand, the approach focuses on the relationship between the lesson preparation (anticipation) and its actual implementation (adaptation). On the other hand, it directs attention to the management of uncertainties inherent to such an environment. These uncertainties are due to the fact that the students' activity cannot be completely predicted, and the teacher is often in an improvisation mode. The conceptual constructs introduced in this frame aim at analyzing the impact of the dynamics of students' interactions with technology tools on the management of the planned (by the teacher) 'cognitive route' (Robert and Rogalski 2005) and the possible divergences from this intended path during the lesson (Abboud-Blanchard and Rogalski 2017).

The teacher's conceptions of the mathematical notion to be taught and of the relation students have to it, are subjective determinants of his/her professional activity. They condition the didactical process that the teacher wants the students to follow, as well as the management of the processes developed during the lesson (Robert and Rogalski 2005). Although the didactic scenario is familiar, the students' diversity and the specific context of the class introduce a factor of uncertainty. In addition, when students are working with a technological tool, the teacher encounters difficulties to control the tool's feedback (which is strongly dependent on students' manipulations) and to identify the interpretations students are making. Teachers have often to deal with tensions due to the presence of the tool and its role in the student's activity but also its interaction with the mathematical knowledge at stake.

Following Rabardel's (2002) *Instrumental Approach*, technological tools could be seen both from the teacher and the students' perspectives. In both cases, the subject-object interactions are mediated by the tool. Nevertheless, the object of teacher's activity is the students' learning, whereas the object of the students' activity is the content of the task given by the teacher; their instruments based on the same tool are thus different. Figure 1 presents how these two instrumented activities are articulated within the dynamics of class preparation.

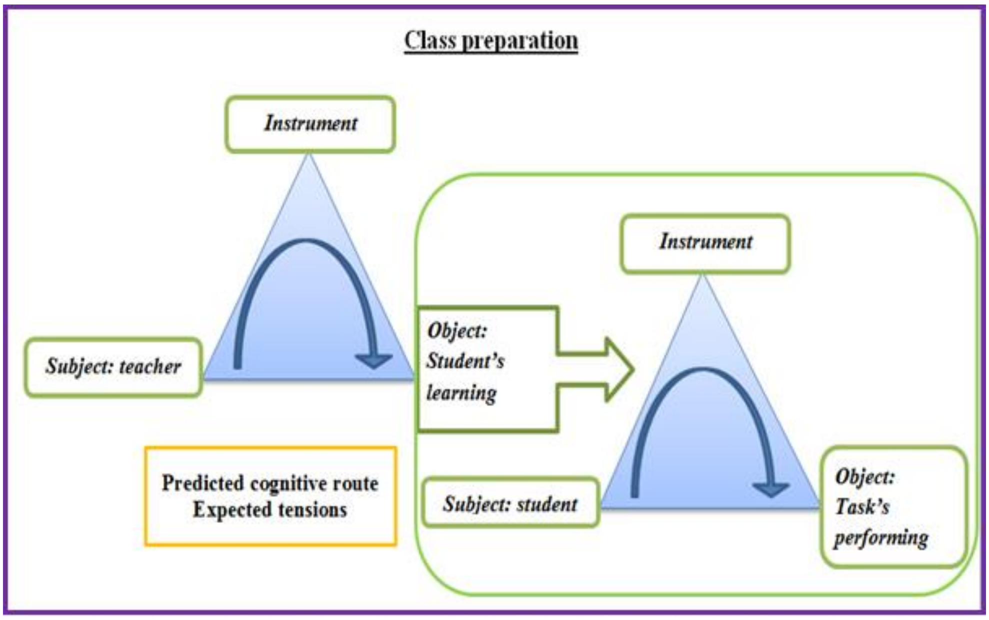


Figure 1: Articulation of the teacher and students' instrumented activities within the preparatory phase

The scene is completed when the two instrumental situations are articulated within the dynamics of class management and indicates possible tensions and disturbances. This is presented in Figure 2.

Tensions and disturbances

In the French approach, there is departure from the way Kaptelin and Nardi (2012) introduced the terms *tension* and *disturbance* when presenting the concept of contradiction, central in Engeström's framework of analysis of how activity systems develop (Engeström 2008). These terms appear in their familiar use; emphasis being put on the analysis of contradictions in activity systems, as key learning sources.

Tensions are not necessarily conflicts or contradictions. In the teacher's activity, tensions are manifestations of 'struggles' between maintaining the intended cognitive route and adapting to phenomena linked to the dynamics of the class situation. Some of these tensions might be predicted by the teacher and so there might be plans of how to manage them. Others are unexpected and constrain the teacher to make decisions, in situ, that direct their actual activity.

Disturbances are consequences of non-managed or ill-managed tensions that lead to an exit out of the intended cognitive route. Disturbances happen when a new issue emerges and is managed while the current issue is not completely treated or when the statement of a new issue is not part of the initial cognitive route.

Here the focus is on tensions and disturbances related to the local level of a class-session; other tensions are or might be managed at a more global level (over several sessions). As indicated in Figure 2, tensions could be related to different poles of the system of teacher-and-student activities; they can be shaped differently along three dimensions (previously introduced by Abboud-Blanchard 2014): temporal, cognitive, and pragmatic.

Tensions related to a cognitive dimension appear in the gap between the mathematical knowledge the teacher anticipated to be used during task performing and those really involved when students identify and interpret instrument feedbacks. Tensions related to both pragmatic and cognitive dimensions are produced by the illusion that mathematical objects and operations implemented in the software are sufficiently close to those in paper-and-pencil context (we refer to Balacheff's (1994) analysis of the *transposition informatique*[2]). Tensions related to a temporal dimension are frequent in ICT environments and linked to the discrepancy between the predicted duration of students' activity and the real time they need to perform the task. Teachers are generally aware of such tensions; they often manage them by taking control of the situation, either by directly giving the expected answer or by manipulating the software themselves.

[2] Balacheff defines this transposition as the process through which the mathematical knowledge to be taught is fundamentally transformed within a computer-based learning environment.

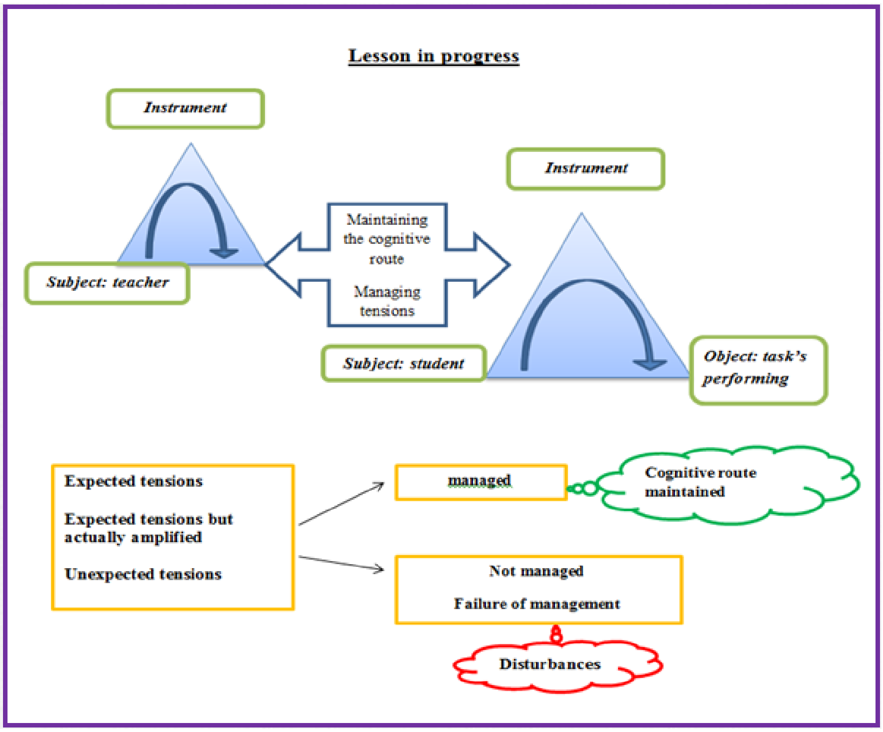


Figure 2: Tensions and disturbances within the dynamics of class management

Finally, a tension non-specific to the ICT environment may concern the didactical contract; students cannot identify the type of answer the teacher is expecting. ICT environments may amplify this type of tension when students are uncertain of the goal of the activity i.e. is the goal about a mathematical object to manipulate with the software or about the use of the software itself?

1.2 Methodological choices

The concern is to analyze the everyday practices of regular teachers who are not involved in research projects and experimental work. The use of technologies these teachers develop and integrate into the day-to-day activity is our actual research object. The choice of data gathering is made to reduce as far as possible the impact of researchers (observers) on the teachers and students' activity in the class. Hence the analyzed sessions are chosen and recorded by the teachers themselves. Deferred interviews and preparation documents are collected in order to identify personal and social determinants of the practices. Comparing the observed succession of episodes with the planned cognitive route, enable detection of tensions and disturbances. The analysis of the practices' determinants makes it possible to shed light on reasons of some of these tensions and on the ways the teacher manages them.

The case study that follows illustrates the variety of tensions, and the management of tensions, which range from routine-based treatments to the non-perception of tensions (Abboud-Blanchard 2015). The latter could entail students getting completely out of the cognitive route without the teacher being aware of this phenomenon. The identification of practices' determinants provides useful information to interpret these outcomes.

1.3 A case study

In this case study, the teacher investigated is Daniel (pseudonym), an experienced (10-year career) secondary mathematics teacher. He was chosen because, on the one hand, he is not involved in any experimental project and is not a technology-expert while, on the other hand, he supports the use of technology in mathematics education. Daniel's interview focused on his teaching experience, the professional context in which he is working, his use of institutional resources (curriculum, textbooks, academic websites, etc.), and on how and under what conditions he integrates technologies into his practices. Daniel chose a geometry session where he uses DGE. In addition of the video-recording, he provided a document explaining the choices made and rationales for the students' task in this lesson.

Summarizing the session

The lesson was an 8th grade (13-14 years) class in a computer room with a data projector screen on which the teacher's computer was displayed. The students were asked to download a file previously prepared by the teacher. When opening the DGE file, students discovered the screen shown in Figure 3.

The teacher then gave a preliminary remark: "please recall that every representation (on paper or computer) of a geometrical figure is inexact; measures given by DGE are approximate values".

Students were first asked to move point M in order to have both triangles AOM and BOM become isosceles at O. Second, they had to find other positions of M satisfying this condition, to observe the AMB triangle and to make a conjecture about the M angle. Last, they had to prove this conjecture, without any further indications of how, and if or not, the computer should still be used.

Approximately midway through the lesson, the students were still trying (or succeeding for some of them) to have angles A, B and the two marked M angles equal to 45°. The teacher made several individual and collective interventions: "You charge yourself with supplementary constraints, so it is difficult to find several positions". Finally, after a 'correct' example was proposed by a student on the projected screen, the teacher showed several cases where the two triangles were isosceles without having OM perpendicular to AB. Over the ten minutes that followed, the teacher moved from the demand of finding more than one (general) configuration to finding “the maximum [number of positions]” for M, and then he asked for “all possible positions”.

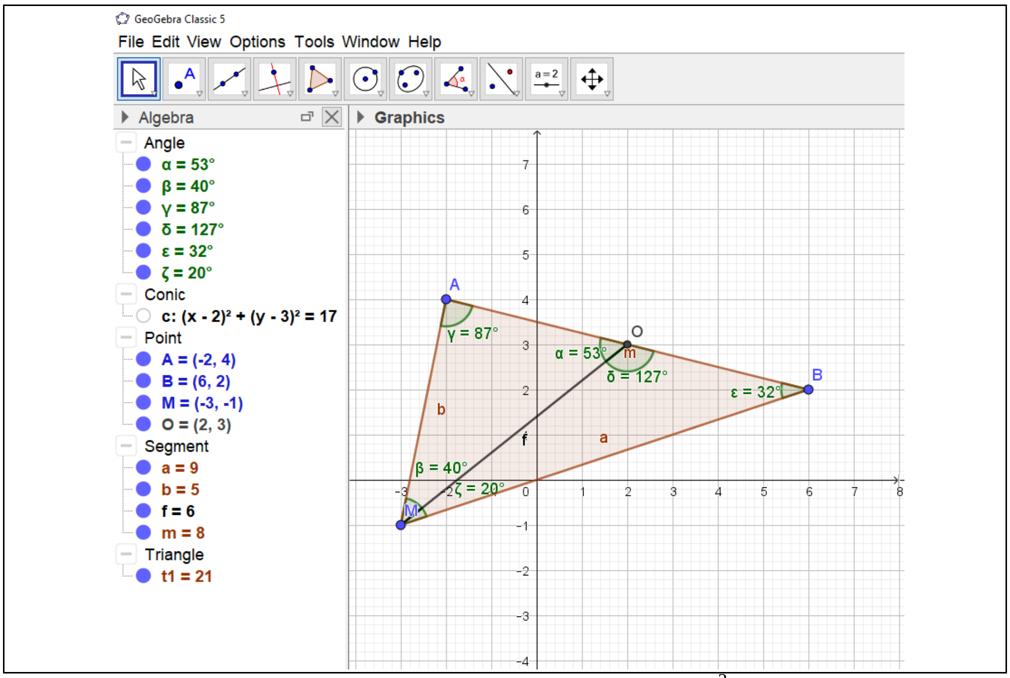


Figure 3: Students’ computer screen[3]

From that moment, the teacher’s goal changed. Rather than discussing students’ responses to this task, he moved to focus exclusively on his overarching goal to establish that the locus of all possible positions was a circle. When a student proposed this idea, he immediately approved and drew the circle and placed M on it. It is only after this episode that he got back to the earlier conjecture and (re)formulated a student’s proposal: “it is always a right angle, yes; that is, the triangle seems to always be a right-angled triangle at M”. He decided then to dictate the present state of shared or, supposedly shared, knowledge; that is to say, the locus of all possible positions of M forms a circle. He postponed the proof of this conjecture because it was already the end of the lesson.

[3] The obvious discrepancy between the displayed angle measurements and the real angle values (for the shapes as defined by the points’ coordinates) is due to the teacher’s choice of rounding for the measured units.

1.4 Analyzing the teacher's choices

What was at stake in this lesson was the theorem related to the circumscribed circle of a right-angled triangle. The presence here of a dynamic geometry software allowed a process of investigation that is difficult to achieve in paper-and-pencil environment. The task was meant to be an introductory task, and not a task requiring a functional use of this theorem in problem solving. It is an unusual task concerning this theorem in the French curriculum and textbooks. Two main choices seem to have been made by the teacher when preparing the task. The first choice was to construct the DGE figure himself and to let students only download the corresponding file. Such a choice limits the instrumented students' activity as concerning the construction of figures through DGE. The second choice was related to what was made visible to students on the screen; in the graphical window, he indicated the angle measures (another choice in the DGE options is to round measures up to units).

Daniel explained these choices by the fact that his aim was to bring students directly to the mathematics exploration of the figure and not to spend time doing it. This was thus meant to restrain the students' instrumented action (limited to handling skills) and to focus their attention on the geometrical exploration and to devote more time to the process of conjecture validation involved in the last question. Unfortunately, (for Daniel), moving M in such a way that the angles become equal is not so straight-forward a task. First, the coordination between observing the angle measurements and moving the point is somewhat complex. Second, the teacher wanted students to focus on angles measures in the graphical window, while several students were focusing on the side lengths in the algebraic window (a cognitive tension that is expanded upon below).

Identifying tensions and analyzing their impact

A pragmatic tension is related to what the teacher expected from the use of DGE and how students actually used it. A part of this tension was indeed predicted by Daniel. That explains the choices he made when preparing the task (see above) in order to minimize the impact of this tension. Yet other parts had not been predicted and these necessitated the teacher's specific interventions. For example, students tried to move points A and B to positions that Daniel did not welcome. DGE allows this manipulation, and students thought that searching for isosceles triangles might be easier if they moved not only M but also A and B. The teacher intervened throughout the session to explicitly forbid many students from moving A and B (given that the teacher did not define them through *Fix object*, within the file initially prepared). When a student was still moving the two points half an hour after the beginning of the lesson, Daniel took control of the computer himself, reset to the initial state and re-explained the task to the student. This is evidence of another pragmatic tension due to students working at different paces through the task, which is often noticed in technology-based lessons and could be highlighted as a characteristic of such a context (Abboud-Blanchard 2014).

A cognitive tension is due to the fact that the teacher considered that the isosceles character of a triangle can be treated by the students both from the angles' property and from the sides' property. Yet, in earlier teaching, the isosceles triangle was defined by the equality of sides – with the equality of angles only having the status of a property. In fact, some students moved the point M by trying to obtain the equality of the sides OM, OA and OB (without controlling the variation of the angle measurements). Daniel was not aware of this and this led to a misunderstanding and even a disturbance for some students. For example, a student encountered the following phenomenon: in the OBM triangle, angle B and M were not equal (45°; 46°), whereas the sides OM and OB were equal (2; 2). The teacher, focusing on the angles (not seeing the sides values) reacted by saying that the equality must be more precise. The student mumbled after the teacher moved away: "I don't understand… it is precise!".

Another cognitive tension linked also to the question of precision goes through the session: Daniel aimed at the continuous objects of (theoretical) geometry, whereas using a software necessarily discretizes them. He also considered that DGE provides approximate mathematical information while students considered DGE information as reliable. This was strengthened by the fact that Daniel rounded all measures to units. This tension provoked several interventions (collective or individual): the teacher reminded students that they must not forget the "approximate character" of what they saw on the computer screen and at the same time he asked them to use what they saw to make conjectures. By rounding to units, there is a finite number of possible positions of M, where there is angles equality. When Daniel changed the initial task by adding a sub-task aiming to find the "set" of all possible positions of point M, he had to state that even if DGE gives a limited number of such positions, there are actually infinitely many such positions. He hastened to bring an end to this contradiction by immediately drawing the circle.

A major temporal tension occurred due to the gap between the planned time for the instrumental task (an average of one third of the total duration of the lesson) and the actual time this task took during the progress of the lesson. Two thirds through the lesson, students were still trying to find several positions of M so as to make a conjecture about the angle AMB. Being aware of the slow progress of the students' activity, Daniel decided to interrupt them and called for a "first assessing" where he gave the correct answers and dictated the conjecture, thereby ending the instrumental task.

The resultant of the set of tensions was thus a major disturbance; in this lesson he had to abandon the aim of engaging the students in an angle-based proof.

Inferring determinants of the teacher's activity

The activity of the teacher was determined by different combined factors. The analysis of the lesson as it progressed enables us to infer the impact and articulation of these factors. The analysis of his post-lesson interview responses indicates particularly personal and institutional determinants.

First, managing conjectures in an investigation process is promoted by the mathematics curriculum for the French lower secondary school (6th to 9th grade). The curriculum also promotes the use of dynamic geometry software for constructing figures and investigating them. Daniel explains his choice of this particular task by referring to these institutional determinants. A plausible inference is that he was expecting (and hoping) that students would engage with a relatively new geometrical topic in an investigative way.

Second, there was evidence of interactions between personal and social/institutional determinants. Daniel chose to present information about the measures of the angles of the 'to-be' isosceles triangles and not about the lengths of their sides; this is unusual. However, starting from the measures of angles allows one to validate the conjecture that the angle AMB can be computed and shown equal to 90°, using the theorem of the sum of the angles of a triangle, something already known by the students, and the fact – implied by the design of AMB – that angle M is composed of two angles, equal to the other angles of AMB.

Third, the use of DGE impinges on Daniel's will to modify and develop his teaching practices (personal determinant). He sees this lesson as an opportunity to introduce a new way for teaching the geometrical chapter devoted to the circumscribed circle and the right-angled triangle by using an innovative task promoted by professional literature (Soury-Lavergne 2011).

Finally, there is the personal determinant of 'being rigorous'. Here, the teacher's choice (about the angles) opens the possibility for a real mathematical proof of the central property about the right-angled triangle and the circumscribed circle - using wide-scope knowledge, instead of referring to figural properties of the rectangle (drawn on AMB by a central symmetry). In fact, during the lesson, Daniel frequently employed logical connectors (so, because, as, then…) in his discourse. An interpretation of this observation, along with considering his will to let students spend more time on the proof process, is that Daniel is strongly oriented toward students developing a logical treatment of mathematical tasks. Such an orientation seems to be a personal determinant of his choice in this particular task.

2. Characterizing teachers' classroom experiences with dynamic geometry technology: An example from England

2.1. Theoretical approach

The complete research study from which this example has been selected aimed to expand knowledge of *how* secondary school mathematics teachers learn through their classrooms experiences to appropriate new technological tools in their teaching (see Clark-Wilson 2010a; 2010b). Whilst the existing research had categorized aspects of teachers' classroom practices (Noss, Sutherland and Hoyles 1991; Ruthven and Hennessey 2002; Drijvers et al. 2010) there was no research that shed light on how these practices had evolved. Initially, Verillon and Rabardel's (1995) theory of *Instrumented Activity* was adopted to gain insights into the nature of the interactions between the *Subject* (here the research lens was firmly trained on the teacher), the *Instrument* (the chosen technological tool) and the *Object* of the activity (the teaching of an aspect of school mathematics to a group of students).

Teachers' professional development is conceptualized as that of 'situated learning' as it is anticipated that the teacher develops their professional knowledge 'in and through' their classroom practice (Lave 1988). This professional knowledge spans the subject at stake (i.e. mathematics), how it is best taught and learnt, which resources might support this alongside institutional knowledge of the curriculum and its assessment.

Following the analyses of sixty-six lessons taught by a cohort of fifteen teachers over a period of a school year, it became apparent that teachers were repeatedly reporting (in their post-lesson reflections) incidents from their classroom that they had not anticipated in their initial lesson design. These *hiccups*, are defined as "the perturbations experienced by the teachers during the lesson, triggered by the use of the technology that seemed to illuminate discontinuities in their knowledge and offer opportunities for the teachers' epistemological development" (Clark-Wilson 2010 p. 138). Key to this definition is that the teacher must have noticed the hiccup. A second phase of research involved the analysis of fourteen lesson observations (of two teachers) that yielded a total of 63 hiccups. The cross-case analysis of these hiccups using a constant comparison methodology led to the classification of seven underlying triggers, as follows.

1. Aspects of the initial task design, such as a poor choice of initial example or subsequent sequencing of examples is unclear/inappropriate.
2. Interpretations of the mathematical generality under scrutiny. Difficulties encountered when relating specific cases to the wider generality (or students failing to notice the generality).

3. Unanticipated student responses as a result of using the technology. Students develop their own instrument utilization schemes for the activity.
4. Perturbations experienced by students as a result of the representational outputs of the technology. Students doubt the 'authority' of the technology.
5. Instrumentation issues experienced *by students* whilst actively engaging with the technology. Students are unclear exactly how to grab and drag dynamic objects.
6. Instrumentation issues experienced *by teachers* whilst actively engaging with the technology. Teacher is unsure how to display a particular representation, i.e. displaying the function table for a given function.
7. Unavoidable technical issues[4]. Displaying the teacher's software or handheld screen to the class.

If lesson hiccups are to be interpreted as a vital contributory element of teachers' situated professional learning as they appropriate mathematical technologies, it is necessary to describe how the hiccup prompted specific aspects of this learning. What follows is a case study of a particular classroom teaching sequence from the research data to justify the *hiccup* as an epistemological construct.

2.2. Methodological choices

Central to the research methodology was the need to observe closely the teachers' development, enactment and reflections on their lesson tasks and approaches in their classrooms through an ethnographic approach. Consequently, a close professional relationship was developed with the teachers such that they felt sufficiently confident for the researcher (Alison) to observe and video-record their teaching, participate in interviews and post-lesson exchanges. The teachers shared their lesson design artefacts (software files, presentation slides, written plan, students' work) in advance of the lesson and, following teaching, produced a written reflection of their teaching, which often included a redesigned task.

2.3. A case study

This example is taken from an English classroom and features an experienced teacher, Tim (pseudonym) and his class of fifteen 14-15 year olds who were being introduced to Pythagoras Theorem through a dynamic task mediated by the

[4] During the study, the teachers were using prototype classroom network technology. This did result in some equipment failures during some lessons. Although these occurrences were definitely classed as hiccups, they were considered to be outside of the domain of the research study as they were not related to the mathematics being taught and learnt.

geometry environment of their handheld devices that were wirelessly connected using classroom management software. Tim had written the following mathematical objective for the lesson, “to appreciate Pythagoras’ Theorem, in particular recognizing that the sum of the areas of the squares on the two smaller sides will equal the area on the longer side *if and only if*[5] the triangle is right-angled”. Furthermore, Tim added a specific intention for the use of the wireless classroom network (that connected all of the students’ devices to the teacher’s computer/projector), “Each individual student will explore the triangles on their own handheld – we will use the shared space of screen capture to come to a shared agreement about the necessity for the triangle to be right-angled”.

The task, which was wholly conceived and designed by Tim based on his a priori analysis of what the students were required to understand, included a software file that was transferred in advance to the students’ handheld devices. The task is shown in Table 1.

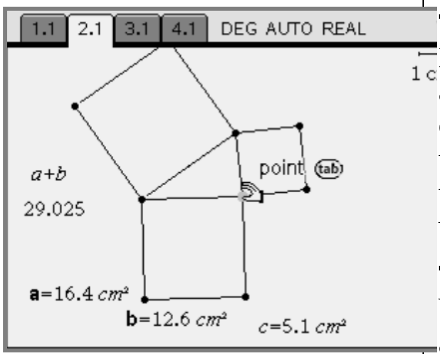
Opening screen on students’ handheld devices	**Description of the construction of the environment**
	The task was constructed in the TI-Nspire ‘graphs and geometry’ application. A triangle had been constructed onto the sides of which three squares had been defined. The triangle was not constrained in any way. The areas of the squares *a*, *b* and *c* had been measured. *a* and *b* were defined as variables so that the value of *a*+*b* could be calculated and displayed on the screen.

Table 1: Tim’s task ‘Pythagoras exploration’.

Summarizing the session

Tim displayed the opening screen of the task and, following a brief introduction to connect the image to some work pupils had encountered previously, Tim then moved the triangles around by dragging different vertices, highlighting which area measurement related to which square. He then stated the aim for the task, which was to move the vertices of the triangle until the area measurements that had been

[5] Tim’s emphasis.

labelled *a* and *b*, when summed, equaled the area measurement that had been labelled *c* saying,

> "So, you need to think about which square is which and move them around a bit and I want *a* and *b* to add up to make *c*. Do you kind of get what we have to do? You're trying to change the sides so that *a* and *b* adds to make *c*."

At this stage Tim gave the students five minutes to respond to this challenge, during which time he moved around the room supporting students and monitoring their work. Simultaneously, the students' handheld screens were on public display to the class, refreshing automatically every thirty seconds. In this period Tim chose to send one student's work (Student A) to the teacher's computer, which captured the student's response to the task at that point in the lesson (see Figure 4). Tim concluded this phase of the lesson by alerting the students that they were going to be stopping and reviewing the class display of the individual handheld screens in a few minutes and that they would, 'scroll down and have a little chat about them [the screens] and see how we're getting on'. With the students' attentions back on the screen capture view of their work, Tim began to pick out screens and check that the numbers displayed satisfied the desired condition by talking out loud. For example, he focused on Student A's screen, saying: "Okay I'll go through these and we'll have a look at them so... *a* add *b* is twenty-eight point six-ish, and there's *a* and *b* is two point five, add them together that's kind of alright – that's really good."

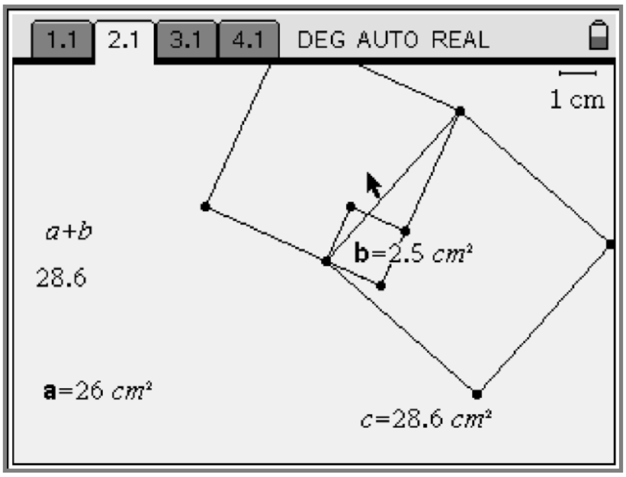


Figure 4: Student A's handheld screen.

He then moved on to Student B's screen, shown in Figure 5, saying: "We've got this one here and we've got three and twelve, that's fifteen and that's nineteen so that's close but a little bit off but it's close".

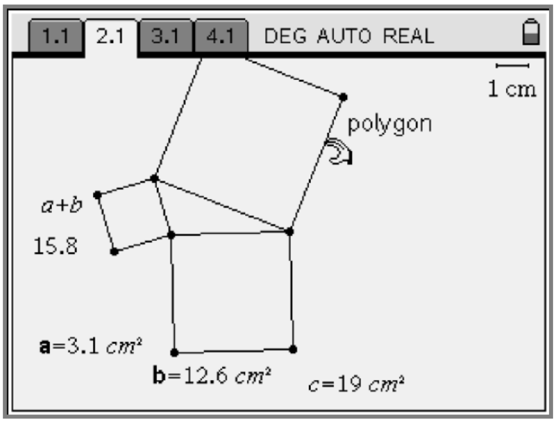


Figure 5: Student B's handheld screen.

At this point he reminded the students that "...we're kind of looking at the ones that do work and the ones that don't..." and he invited the students to volunteer their screen number if they thought that their screen 'worked'. At this point, there was a noticeable increase in students' participation and involvement as a number of students were heard to call out 'mine works', '22 works' and 'mine's 12' and Tim tried to locate these screens and move them so that they were visible to the class.

Tim then directed the students by saying,

> "Okay I'd like you to look at the ones that work that we've identified and compare them with the ones that don't work and I want you to look at the shape of the triangle... ...in the middle. This is what I am asking you to look at now. Look at the shape of the triangle. Look at the ones that work, look at the ones that don't work and my question to you and you've thirty seconds to discuss this now, my question to you is: is there anything different about the shape of that triangle in the ones that work compared to the ones that don't quite work? You've got 30 seconds to talk about it."

After a short period of pupil discussion Tim asked if anyone had noticed anything and a student volunteered the response, 'Is it right-angled?'.

Tim responded by displaying the student C's handheld screen shown in Figure 6 and making the following comment, directed towards the author of the screen:

> "Yours is quite easy to see isn't it? - that this is a right-angled triangle because you've actually got a square and you can see it's a corner of a square in there – yes it is a right-angled triangle".

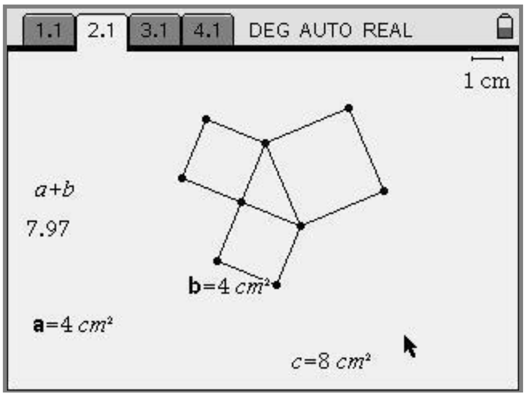


Figure 6: Student C's handheld screen.

Tim then selected a student's screen that did not appear to satisfy the initial task instruction that the value of a and the value of *b* should sum to give the area measurement of *c*, but did appear to work visually in that it appeared that the central triangle was right-angled (see student D's handheld screen in Figure 7).

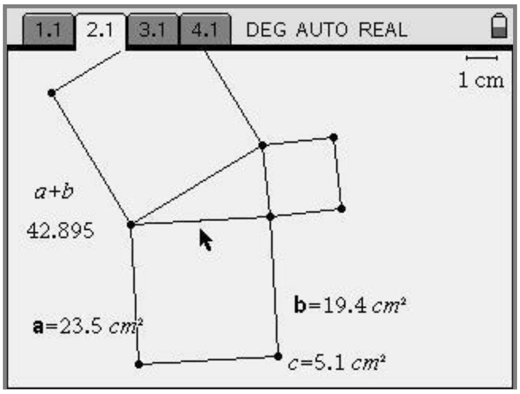


Figure 7: Student D's handheld screen.

Tim said:

> "This one, we've got a add b... doesn't quite make c either, but yours kind of works the other way round, if we look at this square here, that's five, and this square here is about nineteen and five and nineteen is about twenty four-ish and that's twenty four – so yours works a different way around".

Tim selected two more examples and spoke his thoughts out loud to reason through the calculation of the sum of the measured areas to verify whether they did or did not meet the initial task constraint He then asked the students to make a conjecture by saying, ‘So what do you think we are learning from this then? What do you think we are noticing about the ones that work and about the ones that don’t work?’.

One student responded, ‘The more the equal they get then... you know...’ to which Tim requested the student to ‘say that mathematically?’. The student added, ‘They’ve all got a right angle in them’. Tim then prompted the students by saying ‘So if the two small areas make the bigger area...’, leading to the same student’s response ‘it makes a right angle’.

Tim concludes this teaching sequence by consolidating his key learning objective thus, ‘Okay, so that’s what we’re learning here if the two smaller areas of our squares make the bigger area then we… it’s a right-angled triangle. If it’s a right-angled triangle, then the two smaller areas - of the squares - make… the biggest area’.

2.4. Analyzing the teacher’s choices

Here, the focus is the choices made by the teacher in planning and implementing the lesson. Central to Tim’s design was the intention to explore the regularity and generality of the mathematical context provided by a dynamic construction of squares on each of the sides of a triangle. In this task, he had interpreted the notion of the variables a, b and c as the registers of memory of the measured values of the areas of the squares. Tim was explicit in directing the students to change the various parameters within each of the environments, by the dragging of free vertices, with a view to students arriving at their own example that satisfied the constraint that the sum of the two areas labelled a and b should equal the measured value of the area labelled c.

From planning through to classroom enactment, it was clear that Tim set an expectation that the students would arrive at their own interpretations of the generality under exploration, although Tim did take the lead in the selection of the screens that would be discussed. A discourse analysis of the lesson transcript evidenced that, on five separate occasions during the whole class discourse, he was encouraging the students to focus on aspects of the similarity and difference between the properties of the central triangle when the areas of the two smaller squares did, or did not, sum to equal the area of the third square. Early on in this discourse, Tim introduced the notion of ‘it not quite working yet’ to describe a student’s screen where the condition was not met and later on in the discourse, Tim explicitly asks the students to focus on ‘the ones that work’.

2.5. Evidence of a hiccup

What follows is a detailed analysis of one particular hiccup that took place during the lesson in order to show how this event *may* have contributed towards Tim's situated learning during and soon after the lesson.

The hiccup was observed during a point in the lesson when Tim was clearly reflecting deeply on the students' contributions to the shared learning space and 'thinking on his feet' with respect to responding to these. It coincided with his observation of an unanticipated student response. The particular hiccup occurred when a student had found a correct situation for the task; that is, the two smaller squares' areas summed to give the area of the larger square, but the situation did not meet Tim's activity constraint of a + b = c. This hiccup can be classified as Type 1 as Tim's initial task design made it difficult for pupils to identify which of the measured areas (a, b and c) referred to which of the three squares on the screen.

Tim commented about this in his personal written reflection after the lesson:

> "One student had created a triangle for which a+b did not equal c, but (I think) a+c=b. This was also right-angled. This was an interesting case because it demonstrated that the 'order' did not matter... when the sum of the smaller squares equaled that of the larger square, then the triangle became right-angled".

Tim revised the TI-Nspire file after the lesson, providing some convincing evidence of his learning as a result of the use of the technology in that he intended to do something different next time. Tim gives an insight into his learning through his suggestions as to how he thought that some of these perceived difficulties might be overcome by some amendments to the original file.

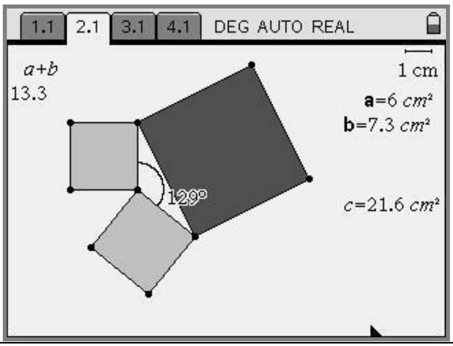
Opening screen	Revisions to the construction of the environment
	Task 2 (revised): The squares whose areas were previously represented by 'a' and 'b' have been lightly shaded and the square represented by the area measurement 'c' has been darkly shaded. Tim also added an angle measurement for the angle that is opposite the side that was intended to represent the hypotenuse.

Table 2: Tim's revisions to the TI-Nspire file for the 'Pythagoras exploration'.

Both of these amendments to the original file suggest that Tim wanted to direct the students' attention more explicitly to the important representational features. He wanted to enable the students to connect the relevant squares to their area measurements and 'notice' more explicitly the condition that when the condition for the areas was met, the angle opposite the hypotenuse would be (close to) a right angle. Here Tim was still trying to overcome the inherent difficulty when using mathematical software concerning the display of measured and calculated values in the hope that students would achieve an example where the areas were equal, and the measured angle showed ninety degrees. This conflicted with his earlier willingness to try to encourage his students to accept an element of mathematical tolerance when working with technology with respect to the concept of equality.

3. Comparing and contrasting the analyses

In the research in the English classroom, the aim was to articulate more deeply the nature of, and processes involved in, teachers' learning as they introduced a multi-representational technology (MRT) into their classroom practices. The identification and analysis of one classroom hiccup, and the identification of Tim's subsequent associated actions, provided evidence of his possible situated learning in relation to the use of the technological tool to privilege students' explorations of variance and invariance. This learning was related to a number of factors.

First, the decision to use the technological environment for this activity, and display the students' results publicly, resulted in an unanticipated student's responses becoming the focus within the classroom discourse. Consequently, Tim was prompted to develop a new repertoire of dialogue in response to this classroom experience that acknowledged the student's correct response within a wider mathematical sense.

A second factor was the design of the task in the technological environment and the way in which its appearance (on the computer screen) would support, or not, students to notice the variant and invariant features relevant to this task. This was achieved by modifying the objects' labelling and introducing a new piece of information (angle measurement) in order to focus students' attention toward the property at the core of the mathematical theorem at stake (in Tim's intention).

Overall, in his original design for this activity, Tim had not envisaged the scenario of the student response that led to this lesson hiccup. The analysis presented of this one lesson hiccup provides an insight into the relationship between Tim's situated learning in the classroom and the potentially more epistemic learning as evidenced by his direct actions in redesigning the activity.

The French analysis focused on the relationship between the lesson preparation and its actual implementation. The focus was on the teacher's management of uncertainties inherent to students' activity in a technological environment; the teacher needs often to be ready to react immediately to students' feedback when working with computers. In Daniel's activity, several types of tensions were observed, manifestations of 'struggles' between maintaining the teaching goals and adapting to the classroom current situation.

Some tensions were anticipated by Daniel, and he planned how he might manage them. Other ones were unexpected or even not consciously noticed by him and not managed within the lesson time: these led to disturbances in the management of the mathematical activity of the students.

The tensions were analyzed along several axes. A pragmatic tension was related to what the teacher, Daniel, expected from the use of DGE and how students actually used it. In addition, cognitive tensions were identified related to definitions of mathematical object or to the intrinsic discontinuity and approximation of measures in the technical environment. Temporal tensions (almost a constant feature of lesson management) were exacerbated by unexpected difficulties, some of which could be due to students' lack of experience in using 'basic' commands of the software.

In this analysis of Daniel's activity, only tensions and disturbances related to the local level of a class-session are considered; some tensions were, or might have been, managed at a more global level (perhaps over several sessions).

Contrasting the English and French studies, a major difference is the positioning of the researchers. In the English study, a close 'insider' relationship was established between the researcher and participating teachers, which required a "theoretically-based, innovative, iterative design process - for reliable developmental outcomes" (Jaworski 2004, p.3). In the French study, the researchers worked on videos of the lesson chosen by the teacher and identified tensions and disturbances from an 'outsider' point of view. While interactions between the researchers and the teacher occurred later, the teacher was not directly involved in the research process and is considered as an 'ordinary' teacher – with the research process aiming, in a way, at some generalization (for activity analysis and for teacher training).

A second contrast relates to the way in which classroom incidents were both identified and theorized. The notion of such 'contingent moments' in mathematics lessons is currently receiving increasing attention in research literature (for example, see the special issue of *Research in Mathematics Education*, Vol. 17, Issue 2, entitled *Tales of the unexpected*). Within the English research, such contingent moments - the hiccups - were conceived as an epistemological construct through which to identify aspects of the teacher's (mathematical) professional learning.

In comparison, the French study conceived the existence of tensions (and the possible disturbances) as inherent to the characteristics of the teaching situation, particularly when involving technological environments - as tools both for the teacher and the students. The research focus was not on evolution in the teacher professional knowledge but on the dynamics of managing tensions, and on the factors that influence this management: on the one hand, it depends on the 'contingencies' in classroom mathematical life and, on the other hand, it is oriented by several forms of determinants of the teacher's activity (from institutional to personal ones).

Conclusion

In comparing and contrasting the analyses, several themes emerged relating to the respective theoretical perspectives, the methodological approaches, the relevant unit of analysis, the research outcomes, and the long-term intentions. In this section, our discussion addresses each of these themes.

In terms of theoretical perspectives, in the English research the notion of the hiccup was employed to articulate teachers' professional learning over time as they integrate dynamic mathematics technology in their lower secondary school mathematics lessons. In the French research, the idea of 'tensions and disturbances' aimed at a better understanding of the issues involved in the integration of dynamic mathematics technology into lower secondary school mathematics lessons by 'ordinary' teachers. Researchers envisage to investigate again (the years to come) the same teacher if the opportunity arises, in order to see the evolution of his practices.

The methodology of the French research entailed lesson analysis based on video recording of the lesson, together with post-lesson interview with the teacher (providing insights into his practices' determinants), analysis of the tasks proposed to the students and how the teacher implemented the tasks in the classroom. In the English research, the methodology entailed pre- and post- lesson interviews, lesson observation (with the lesson audio and video recorded), plus analysis of the lesson artefacts such as the teacher's plan, the software files, the student productions, and so on.

Given the theoretical perspective of the English research, the unit of analysis was the individual teacher's professional learning. Here, the 'grain size' was both 'micro', in terms of detailed analysis of individual hiccups, and 'macro' in terms of identifying teachers' learning trajectories over time with respect to their mathematical, technological and pedagogical knowledge. In the French research, given the theoretical perspective of the research, the unit of analysis was the individual teacher's anticipation and adaptation in implementing their lesson. Here the 'grain size' was 'micro' in terms of detailed analysis of tensions and disturbances, 'meso' in terms of analysis of the teacher's adaptations during the session itself and in a later session, and 'macro' in terms of inferences about the determinants in the teacher's activity.

The longer-term intention of both the English and the French projects was to provide deeper insight into the ways that teachers use mathematical technological tools in their classroom practice so as to inform the design and implementation of professional development activities to this effect. On the English side, it was anticipated that it might be possible to address common types of hiccups within professional tasks for trainee and practicing teachers to promote and encourage reflection in, and through, classroom practice. On the French side, an additional aim was to provide theoretical and methodological tools that can be used in teacher educator courses in order to improve their understanding of the complexity of ordinary teachers' practices related to technology and to adapt, accordingly, their training actions. Both of these research endeavors contribute to the call made by Sinclair and colleagues (2016, p. 704) for "further research on the preparation of teachers [in the use of technology] to help them ensure that students gain deeper understanding of geometrical concepts and theory".

In conclusion, as evidenced by the discussion thoughts we have presented immediately above, the French and English studies provide insights into both sides of the same coin, that of teachers' classroom practices with digital technology in the classroom. In fact, on the one side, the French analysis is particularly oriented toward the reasons producing *tensions* and *disturbances*, on the other, the English analysis emphasizes the consequences of *hiccups* for teacher learning.

Whilst the context ('ordinary classrooms') and overarching longer-term intentions (theorizing about aspects of technology integration) of the two studies are closely aligned, the complexities of technology integration in mathematics lessons are illuminated in ways that explain teachers' decision making both in the moment, and over time. By contrasting the two studies, we have shed light on the many and varied considerations that mathematics teachers face with integrating technology.

Here we have illustrated how the hiccups, tensions and disturbances when integrating digital technology in mathematics classrooms leads to teacher learning. Yet the occurrence of such hiccups, tensions and disturbances can, potentially, put teachers off using digital technology for ever or mean that they do no more than the minimum. Jones (2011, p. 44) has suggested the notion of *canalization*, a term usually used to indicate that there is a 'normal' pathway of development, to capture the idea that when more is known about the complexities of digital technology integration in school mathematics, then technology use "may be more likely to reach a 'tipping point' and move the pathway of education to a radically new route". Our research contributes to a deeper understanding of the complexities of such technology integration in school mathematics towards when there may be such a 'tipping point'.

The theorizing evident in the French and English studies emerged from the analysis of digital technology-rich mathematics classrooms. Nevertheless, the theoretical constructs (*hiccups*, *tensions* and *disturbances*) may well be useful when analyzing lessons where there is no use of digital technology. This needs to be validated by further studies.

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