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THEORY AND THE ROLE OF THE MATHEMATICS TEACHER EDUCATOR: COMPARING THE USE OF VIDEO IN TEACHER EDUCATION SESSIONS IN FRANCE AND ENGLAND

Abstract. In this article, we compare and contrast practice-based approaches to using video in the context of mathematics primary and secondary teacher education. We look across country boundaries, with a focus on theory, in relation to the role of the mathematics teacher educator. We place the article in the context of developing interest in the facilitation of professional learning of mathematics teachers using video. In contrasting our different practices, we ask: what guides the planning of video sessions? what guides the action of facilitators during sessions? and, what are the intentions, in terms of teacher learning? We uncover similarities and differences in our practices which we theorise in terms of our espoused, enacted and intended theories, as mathematics teacher educators.

Keywords. Using video, teacher educator knowledge, role of theory, mathematics teacher training

Résumé. Une comparaison de l'utilisation de vidéos en France et au Royaume-Uni pour la formation des enseignants de mathématiques : théories et rôle du formateur. Dans cet article, nous comparons des pratiques effectives d'utilisation de la vidéo en formation d'enseignants du 1er et 2nd degré, pour l'enseignement des mathématiques. Au-delà des différences culturelles, nous nous interrogeons sur le rôle joué par la théorie dans ces approches, et nous nous inscrivons dans le courant de recherche actuel sur le rôle du formateur d'enseignants de mathématiques. En comparant nos pratiques, nous nous demandons ce qui guide l'organisation de la formation et l'action du formateur pendant ces séances utilisant des vidéos : quels sont les enjeux, en termes de développement professionnel, pour les enseignants ? Nous mettons en lumière les similarités et les différences dans nos pratiques, que nous analysons à travers l'idée de « théories du formateur », explicites ou non, transmises ou non aux enseignants formés.

Mots-clés. Vidéo, connaissances professionnelles pour la formation, rôle de la théorie, formation des enseignants en mathématiques

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Introduction

We came to write this article through our participation in two Symposia focused on making connections between English and French approaches to mathematics teacher education, from a theoretical perspective. We place our writing within the field of growing attention paid to the role of the didactician or facilitator of teacher education (e.g. Jaworski and Huang 2014). In this article, we illuminate similarities

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p. 119-144

and differences in the three authors' (all teacher educators) uses of video when working with mathematics teachers. We discuss some cultural differences between our English and French perspectives with respect to teacher education. In particular, we focus on the way that theory informs what we do: what guides the planning of video sessions? what guides the action of facilitators during sessions? and, what are the intentions, in terms of teacher learning?

Jaworski and Huang (2014) proposed the word 'didacticians' as a label for the specific group of mathematics educators involved in the training of pre-service teachers, or in the professional development of in-service teachers. From a French perspective, 'didactician' singles out those educators who are also researchers in mathematics education in distinction to those 'formateurs' who are involved in the education of pre-service and in-service teachers. We use the phrase 'teacher educator' to describe what we do as we are reporting here on our work with mathematics teachers – all three of us are didacticians in the French sense of the word as well.

1. The role of the teacher educator

In a commentary on a journal special issue with the theme of the practices and professional development of teacher educators, Even (2014) suggests that although there is growing interest internationally in the education of teacher educators, there is currently little research addressing this area. Even calls for 'a more comprehensive research effort on the education and professional development' of teacher educators (p.331). We view this article as a contribution to such an effort, as we share what we have learnt from each other's practices and the influences that have led to us acting in the way we do. We are in a peculiar role as teacher educators, in that, through teaching teachers we are enacting and exemplifying in practice what it is to teach, as well as, perhaps, discussing and espousing theoretical perspectives or beliefs about what it is to teach. We recognise that our actions may not always match our expressed beliefs and stances. We also recognise that the learners in our sessions (pre-service or in-service teachers) may pick up more from what we do (in the role of teacher) than from any explicit content. Our concern in this article is with the role of theory in our work as teacher educators. In particular, how do theories of learning and teaching, guide or influence the actions of teacher educators?

We begin by setting out our methodology for comparison of practices (section 2) and then offer a description of theory and practice in one English context (section 3) and theory and practice in two French settings (section 4), before analysing similarities and differences (section 5).

2. A methodology for comparison of practices

The collaboration on which we report in this article is in the tradition of teacher-educator self-study (Loughran 2002; Tzur 2001). We examine our own practices in an effort to understand better what we do and what each other does with a focus, as stated above, on the influence of theory. We are teacher educators and researchers at the same time, although the roles are different, and the decisions we make, knowingly or more implicitly, may depend on which role we assume.

It is one thing to analyze what takes place in a training session or program (with theories as tools for analysis), and another thing to actually teach a session (where we act in-the-moment to make decisions and perhaps only later analyze those decisions in relation to theory). Being an educator as well as a researcher doing research on teacher education, it can be hard to separate our different expertise and practices. Here we analyze sessions that were meant for training and not for an experiment on training, but our two roles are intertwined.

As researchers, we are involved in problematizing the teacher education system, leading us to look at different levels of theory in our field. We identify the following uses of theory in our own work:

- the theories (or elements of theories) that guide the choices made when designing training programs;
- the theories that inspire the effective implementation and actions of teacher education;
- theories that we use to analyze classroom and teacher education activities;
- the elements of theory that might be among the knowing that we intend on offering to teachers.

In essence, we take 'theory' to mean any set of distinctions relevant to one of the purposes above. We conceptualize these different uses of theory under the following headings: espoused, enacted and intended. Our espoused theories are the ones we perhaps write about and use to justify our research and that may inform our planning. We distinguish, however, espoused theories from theories that are enacted. In the performing of a teacher education session there may be a more or less close match between the theories being espoused, those being enacted (and, indeed, those intended for the students to learn). A caricature of a mismatch between espoused/intended and enacted theories would be a lecture given on the importance of learner-centred education. However, some differences between the espoused and enacted theories might be inevitable, since teachers' education and mathematics teaching have their own specificities. We identify the intentions of the teacher educator in relation to the intended theory that will be taken on by the student teachers in a session. There may be an intention for students to espouse the same theories as the teacher educator, or enact the same theories, or something

different. Part of the contribution of this article to the field, is to offer the framework of espoused, enacted and intended theories, and to exemplify its use for the comparison of teacher educator practices, when using video with mathematics teachers.

2.1. Comparing use of video[1]

In the last decade, there has been increasing use of video for teacher education and professional development, across the world and across subjects with, broadly, one of three intentions: linking theory and practice of teaching; analyzing professional practice; and, implementing institutional reforms (Gaudin and Chalies 2015). Our own work is part of this growth and crosses the aims of linking theory to practice and the analysis of practice. A similarity across our three contexts is the use of video as a mechanism to provoke teachers to reflect on practice and a conviction about the need, for effective working with video, to support teachers in moving beyond generalized and evaluative descriptions of what is seen and into a space where it is possible to dwell in the detail of events and allow new perceptions and connections to arise (Coles 2013), even though we do not deal with this need in the same way. We all tend to use local video recordings of teacher practices rather than, for example, internet-available videos that may have been edited for particular purposes.

Despite these similarities, we had the experience of co-running a workshop at a conference in 2016 (ICME 13 in Hamburg) where we tried to find one common lesson video clip we could use with participants, to exemplify our different ways of working. We could not find a suitable one, in terms of mathematical content, of length (a short clip or an entire session), of exhaustiveness (with or without editing), of available information on the context of the video (experience of the teacher, moment in the planning of the year, curriculum) – demonstrating that despite many similarities of aims, we have different expectations when choosing a video for teacher education. We needed an English language video (due to the language of the conference) and the examples Alf had available were not suitable for Aurélie and Julie's purposes. For example, one video Alf has used frequently involves a clip of a class working on the question: "How many numbers are there between zero and one?". Without more elements about the context (what had been studied before, intentions of the teachers…), this question was too broad for the others, and, as will become evident later in the chapter, would not have been a good choice to illustrate the French way of working, which requires an anticipation of possible student answers and how they are put into use in the mathematical

[1] In the present article, we will use this term to mention any video clip extracted from a videotaped session of mathematics teaching.

content at stake. Aurélie and Julie offered a related question, which would have worked if we had a video of a class working on it: "Can you give me a fraction between a third and a half?". We note, in passing, that we learnt more about each other's practices when faced with the practical need to choose a common video than we had in many hours of discussion of our practices prior to that. Part of the problem, we recognized, is that we can use the same word to mean different things and hence interpret what each other is saying through the prism of our own practice. In writing this article, for example, we are aware that we have different connotations for the word 'theory', hence in part wanting to look at theory use as it espoused or enacted and at scales from local theories informing practice to overarching orientations.

In the next two sections, we offer exemplifications of our use of video. These necessarily involve a degree of description, in order to give access to the context of our work. We have chosen to structure what we present in the same way – initially offering a theoretical perspective and some institutional background, then going into the detail of our ways of working, either drawing on a specific example or generalizing across sessions and ending with an articulation of intended outcomes. We then look across these examples in order to highlight similarities and differences in relation to the use of theory. We conclude by returning to three questions that have motivated this article: what guides the planning of video sessions? what guides the action of facilitators during sessions? and, what are the intentions, in terms of teacher learning? We have structured our exemplifications of practices in order to make our comparison, in relation to these questions. There are differing amounts of detail offered about specific sessions and this simply relates to the available data we had in each context.

3. An example of video use in an English setting (Coles)

The practices of English teacher educators have no central or shared theoretical basis. University teacher education courses sometimes have a strong theoretical background, but this tends to be due to the presence of a researcher with a particular perspective (for example the University of Cambridge primary education course makes use of the Knowledge Quartet, which is a set of distinctions derived from that institution, Rowland, Huckstep and Thwaites 2005). So, in this section, I reflect on the use of theory in my own context with no claim to wider generality.

3.1. Overarching theoretical background

Through the influence of Laurinda Brown, the University of Bristol's mathematics education courses are designed from an enactivist perspective (Varela, Thompson, and Rosch 1991). The courses are designed to get novices learning about teaching (or, in master's and PhD work, learning about researching) in the same manner that experts learn, but from the very beginning (see Brown and Coles 2011). The

enactivist thinker, Francisco Varela, recognized a characteristic of expert performance and learning (in any sphere), which is that experts tend to act spontaneously – and in most cases their automatic responses are effective. However, on occasion, their expert functioning breaks down and, in these cases, they are able to reflect on what occurred in a manner that brings to light the 'intelligent awareness' (Varela 1999, p.31) that led to the behavior (that was not effective). Having identified an awareness that in the past has led to effective behavior but now does not, the expert is able to identify what they need to do differently in future in the same circumstances. A clear example of the breakdown in effective behavior occurs when an experienced teacher moves to work in a different school. Years of developed behaviors that are effective in one context may no longer 'work' in the new scenario. Student teachers on English teacher training courses can experience such a change when they change placement schools (at Bristol, teachers have extended placements at two different schools generally, over their training year). For example, in one school, waiting in silence for a class to quieten down may be effective. Changing to a new school, the same strategy may lead to more and more disruption. The expert is characterized not so much by being able to second guess what will be effective in a situation, but by being able to change and adapt quickly.

Varela's insight (1999, p.30) is that even novices can learn like experts. What is needed is to support novices to steer a path between unconscious behaviors and over-deliberate actions. In other words, they need to act spontaneously and then be supported to reconstruct the awareness that led to any ineffective behaviors. It is this insight that informs the teacher training courses at the University of Bristol. From the very start of the year, the student teachers are placed in the role of having to act as 'teacher'. In the first week of the course, they have to teach each other (in groups of around fifteen) something non-mathematical (that they have prepared) for ten minutes. The rest of the group then reflects on what they learnt, or anything that hindered their learning and the student teacher giving the lesson will begin the process of reconstructing what led to ineffective behaviors and therefore developing 'action targets' for the next time they teach.

The overall aim is to induct student teachers into a cycle of reflection which begins with describing the detail of experience; moving to identifying issues arising from their experience; and then, committing to new actions, linked to the issues identified. This cycle is set up in the first week of the course and it informs: all sessions at University; the writing tasks for students; and, the ways in which, as tutors, we run feedback sessions in school after observing them teach.

3.2. Ways of working with video

I was involved in a research project in 1999, as a teacher-researcher that made use of video recordings of lessons both for professional development of teachers and as a tool for analysis. Arising from these experiences, when I became head of a school mathematics department (with responsibility for the professional development of mathematics teachers) I was keen to use video. I encouraged staff to take video recordings of lessons (with a fixed camera on a tripod at the back of the room) and I would use small clips of these recordings to discuss at departmental meeting (see Coles 2013). I moved to a role at the University of Bristol in teacher training in 2010 and have subsequently made use of video in a number of scenarios, for example: on the training course for secondary mathematics teachers; in Master's level sessions on observational methods; at one-off invited sessions with groups of teachers.

I have come to have conviction in the importance of starting work on video with a reconstruction of events. By this, I mean that the initial discussion needs to focus simply on describing what took place. However, focusing on the detail of our observations is, for most people, an unusual experience and can be hard to do, for participants. There is an ambiguity also, in that descriptions can potentially be at any level of detail. It is probably not going to be helpful, if the focus is on learning about teaching, to go into the minute detail of ergonomic movements. The intention, in starting with reconstruction, is to focus discussion on agreeing the words that were said and possibly some basic description of movements.

The practice of starting with a reconstruction of events is a strategy taken directly from the practice of working with teachers on video developed by the Open University in the UK (see Jaworski 1990). There will often be a need, as the teacher educator (in this context, the facilitator of discussion), to impose the discipline of only offering descriptions of events. Particularly if groups are not used to working in this manner, it is common for initial comments to tend to the general and the evaluative (e.g. 'the class seemed bored', 'the teacher had a lovely rapport with the class'). The teacher educator, in these instances, needs to intervene – cut the contributions short if necessary – and re-inforce the discipline of only describing events that can be observed. You cannot 'see' a pupil being 'bored', for example, but only interpret this. The aim of this section of the way of working is to get teachers to put descriptions like 'bored' to one side and focus on what they actually saw (e.g. 'two pupils at the front were looking out of the window').

What Jaworski (1990) reports from her work with teachers on video, is that typically, we respond to video clips of lessons with evaluative comments (e.g. 'my pupils could never do that') and, as a result, little of value comes from discussion. Combining this insight with my enactivist convictions, what is needed is a mechanism to try and get teachers talking (about video) in a non-evaluative manner

so that there is the possibility of 'seeing' what is on the video differently and therefore allowing discussion to throw up the possibility of acting differently. The discipline of starting with a reconstruction of events is one way of cutting out evaluative comments to allow the possibility of new insight.

At some point, I will always re-play the clip or a section of the clip for teachers to watch again. I try to look out for points of difference amongst the group, in terms of what they saw on the video clips, as these points of disagreement (e.g., about what was said, or the order in which it was said) can provide a motivation to watch again. There is always a delicate decision about when to replay a clip. Leave it too long and the reconstruction task turns in to one of memory, and teachers may lose engagement. Replay too soon and teachers may not have an experience of doubt or questioning about their own recollection of events.

When the replaying works well, teachers in the discussion often comment with amazement at how much more they hear in the clip the second or third time around compared to the first time. Particularly if the focus for re-playing is on a small section, it can become clear that whole sections of dialogue were not heard the first time around. This realization in itself can be a powerful learning point from working with video, with the obvious question it raises of how we cope with this complexity in the real time of classroom decision-making.

Having reconstructed the video, with the aid of re-watching, the final part of this method of using video is to move to an interpretation of the events on the clip. It is necessary for the teacher educator to mark this shift in the discussion, i.e., that it is now moving to a discussion of teaching strategies linked to the particular focus of the group. No matter what the focus is, given the work we are discussing here is with teachers, a focus on teaching strategies is relevant. I take a teaching strategy to be anything that a teacher does, and that can be described in a manner that makes it repeatable.

3.3. Institutional background

In England, there are routes into teaching via under-graduate degrees or one-year post-graduate degrees. Post-graduate provision is split between degrees offered by Universities and ones offered by schools with a University partner who accredits the qualification. The context, in 2018, is that there are not enough mathematics teachers at secondary level, partly as a result of high numbers leaving the profession after a few years (Des Clayes 2017). When teachers are in school, continuing professional development opportunities are available from a range of sources, including Universities, government funded « hubs » and private providers. These opportunities could be: ongoing Master's degree courses; other courses that run over time; or one-off conferences or seminars.

3.4. Example of a session

In the session I describe here, I draw on data from a « video club » that I ran for primary mathematics teachers. The session chosen typifies my use of video and was not exceptional but does illustrate the way of working. A group of in-service primary teachers volunteered to join the club, which committed them to attending six meetings (roughly one per fortnight) after the end of the school day. The volunteers knew that they were committing themselves to taking some video recordings of their teaching and sharing these within the group. An open call had been advertised to teachers in the Bristol area and no one who applied was excluded. Eight teachers joined the group and I analyze here audio recordings from the first session with the group.

What is the video and why was it chosen?

The first session of a video club that involves teachers who do not know each other, is the one instance when I will use a video clip that is not from one of their classrooms. In this case I chose one from the Video Mosaic collection (https://videomosaic.org). The way of working necessitates a clip of 3 to 4 minutes. I have always worked with video clips that show a phase of whole class discussion, i.e., where there is one conversation happening in the room (or at least one predominant one) and also where something unexpected (Rowland and Zazkis 2013) occurs, to which the teacher has to adapt.

In the clip I chose (‘Alan’s Infinity’), the teacher (who is in fact a researcher) is working with a class of 4-grade students, and the clip starts with the teacher asking the class “How many numbers are there between zero and one?”. What happens next on the clip is a discussion amongst students in the class, with two boys doing most of the talking, one of whom thinks there will be infinitely many numbers and the other who disagrees. Three other students are also seen to contribute. The teacher makes some prompts and on occasion directs who will speak next.

The activities during the session using video

Before watching the video: in the first group meeting, before watching any video, I invited each teacher to say something about why they had joined the group and what they were wanting to develop in their own teaching. For example, one teacher (J) described wanting to develop his teaching so his students became more independent in their learning. In setting up the first video watching, I explained the question the class had been offered (how many numbers are there, between zero and one?) but we did not work on, or discuss, the question ourselves. I then said to the group:

> ‘Don’t worry about taking any notes. We’re going to watch a short clip. And the first thing we’re going to do as a group is to literally try to reconstruct

what happened, what was said … and then given all the things you're thinking about we then might do some thinking about what the teacher's doing or what reasoning or what teaching strategies; things that might be more of an analysis. But the first bit is going to be literally what was said. So, the children are thinking the problem, how many numbers are there between zero and one?

After this, I played the video and sat down.

During watching the video: I consider the whole process of 'reconstruction' as taking place 'during' watching – in fact, there is a movement between watching and discussing, re-watching and discussing. The dialogue, straight after the video clip ended, was as follows:

P: I can't stop watching thinking about your [looking at Teacher J] independent children and unfortunately the children that weren't paying attention.
J: yeah, yeah, yeah
Alf: So, that's an interpretation and at this stage the invitation is to say what happened, what you saw
N: She invited them to as what's inside that line
Alf: Anyone remember anything before that, so say that again, so he puts his hand up
N: It was about splitting the line into zillionths.

As the facilitator, my role during the reconstruction phase is to direct conversation back into the detail of events and to offer a re-watching of the video, when the group has arrived at conflicting memories of what took place.

P: Someone talked about atoms didn't they?
J: That was when he said about a really long number line.
J: I thought that was interesting because
Alf: That sounds like an interpretation
J: Interpretation, yeah, yeah, yeah.
Alf: Try and stay with detail, we'll go on to that in a second. Let's try and see if we can get the chronology … and we can go back and look, but we got something from the teacher, a possible question, we think
C: How many numbers
Alf: okay
J: How many numbers do you think?

In this transcript, as well as re-emphasizing the need to avoid interpretation, I articulate where, as a group, we have some questions about what took place on the clip – in this case, what the teacher actually said at the start of the clip. Just before I do re-play, I comment:

> Alf: okay, so we can quickly watch it again. There are some questions about this dust particle and what the dust is all about, something about what's said at the very beginning. Okay so let's try it again.

I then offer one further re-watching:

> Alf: So I might stop it after the first break and we can see if anything else has emerged or if we have any answers to those questions.

At each re-watching, we look at a smaller and smaller section – as we focus on specific questions of what took place.

After watching the video: we spent twenty minutes working to reconstruct the clip, re-watching sections of it three times, before we move to the analysis phase. I provoke this new phase as follows, and Teacher P is the first to respond:

> Alf: Any reflections on what the teacher was doing then or what the students were doing, or any teaching strategies?
> P: I thought she was very controlled and very restrained. I talk far too much in my maths lessons I think. She just let them get on with it.
> Alf: Okay [AC writes 'controlled/restrained/let them get on with it']

My aim in this phase is to support teachers to generalize from what they observed, identify issues relevant to their teaching and, if possible, share strategies related to these issues. For example, the issue raised here by Teacher P, I interpret as 'letting [the pupils] get on with it' and we would then share strategies, i.e. things as teachers we can do, which relate to this aim.

Outcomes from the session

The initial focus on a pure reconstruction of events tends to mean the interpretation of events is rich in detail and noticing. As a teacher educator, my aim is to support the articulation of new ways of seeing in the classroom. The move here is away from the fine detail of classroom events, but not to become so abstracted from the context that the link to direct actions is lost. If discussion moves into the realm of philosophy, for example, whether the class acted in an 'autonomous' manner or not, then my sense is that this is unlikely to be of benefit to teaching. There needs to be some abstraction from the detail, but the link to future action is vital.

To sum up, my aim as a teacher educator working with teachers on video of lessons, is to support new ways of seeing what is there on the clip and get to novel (for those teachers involved in discussion) articulations of features of the video clip. I do not use video with the aim of directing discussion onto particular and pre-identified aspects of pedagogy, beyond having in mind the overall focus for meeting, which is usually some aspect of teaching and learning. My belief, born out of the enactivist world-view, is that learning for teachers will be most effective if what arises out of discussion for them to work on has come from their own

awareness. The shift in perspective of Teacher P, from initially seeing children who 'weren't paying attention' to later on getting to a realization that 'I talk far too much in my maths lessons I think', is an example of the potential power of the way of working in terms of shifting participants' attention away from their own immediate reactions (that are often emotional) and towards potential learning points.

3.5. Analysis in relation to theory use

The way of working on video is theory-driven (Jaworski 1990) and although the origins of the method are not enactivist, the principles behind what I do fit well with my espoused enactivist principles. These espoused principles and theories are not part of the training sessions using video. What is made explicit is the distinction between observation and interpretation, which is an important element in the discipline of noticing (Mason 2002) and features in enactivism (Maturana and Varela 1987). The intended theories, in relation to the teachers, are two-fold. There is an intention that teachers will become conscious of the observation/interpretation distinction; secondly, the hope is that teachers will find 'issues' (for example, for Teacher P, perhaps 'letting the pupils get on with it') that will inform new actions in the classroom. I might describe such issues as local 'conjectures' about practice, or local 'theorising', mindful that from a French perspective 'theory' denotes sets of ideas that are far more developed and established.

4. Two examples of video use from a French perspective (Chesnais and Horoks)

First, this is our personal French perspective, rather than one that could represent every French teacher educator's view on teacher education, and it is mainly inspired by the frameworks we use while doing research about teachers' practices. If we tried to analyze education programs for teachers in other universities in France or observe and analyze what educators do, we would probably see that there is a wide variety of practices, some inspired by other theories, even outside of the mathematics education field, but also from former experience as teachers, as in Sayac (2013), who explores the practices of teacher educators from different backgrounds.

4.1. Overarching theoretical background

To analyze and interpret teachers' practices, but also to consider our own practices as educators, we use the Theory of the didactic and ergonomic Double Approach (Robert and Hache 2013), which combines didactic analyses of pupils' mathematical activities with ergonomic analyses inspired by the analysis of the practices of a professional activity. The fact that the Double Approach was inspired

by activity theory plays an important role in our choices. The main postulate of these frameworks is that teaching practices (teachers' activity) influences pupils' activity, which is responsible for pupils' learning. It allows us to take into account some constraints of the profession, which can explain some of the decisions made by a teacher (or a teacher educator) when teaching (or training), by defining five components of teachers' practices (see chapter 3). The first two concern what happens in the classroom:

1. the cognitive component "corresponds to a teacher's decisions regarding content and tasks, including their organization, their quantity, their order, their inclusion within a curriculum beyond the class period, and plans for managing the class period". (Robert and Hache 2013, p.51);
2. the mediatory component describes choices regarding class events, and the effective implementation in class of the content and tasks (teacher's speech, pupils' participation, assistance to pupils, validations and explanations of knowledge).

The other three components might have an influence on what happens in the classroom, but depend on factors outside of the classroom, such as the professional environment:

3. the personal component (including representations, knowledge, experience of the teacher);
4. the institutional constraints (related to the nature of the mathematics to be taught, curricula, the schedules, the resources available, the administration and inspections);
5. the social constraints (resulting from the various groups formed by pupils, parents, colleagues...).

We divide and analyse the complex system of a teacher's practices into these five deeply intertwined components, which allows us to try to understand the rationale behind a teacher's actions, regularity and coherence relating to his/her decisions for a class. Some of our hypotheses about teacher training come from the Double Approach: taking into account the constraints of training and teaching (for example the fact that not everything is possible for any teacher in any classroom, and also that the teacher is not alone in his or her classroom or in the institution) and, taking into consideration the actual practices and needs of the teachers during training. This is why we believe in the use of videotapes and "the collective discussions about practices, using a professional vocabulary which will help the participants with the necessary ''depersonalization'' in order to achieve a scientific debate, rather than an ideological one" (Horoks and Robert 2007). Videos seem to be a good tool to get an insight into teachers' and pupils' activities and access the complexity of the teaching-learning process, without having to take the risks inherent in actually running the class. For example, it allows us to work on two

components simultaneously: the cognitive and mediative ones, since the video shows both lesson content and ways of managing the class. It also allows us to approach the multidimensionality of teachers' activity and the experienced needs of teachers. Videos also inform about pupils' activity, since we have access to some of their questions, comments or discussions, and to what the teacher says during the pupils' activities (potentially influencing them) and about them. Hence, analysing videos can contribute to a better comprehension of the links between teaching practices and pupils' activities.

Our theoretical approach on practices and their development, encourages us to try to foster their evolution in a bottom-up process. Instead of studying the content first with student-teachers, then elaborating tasks and reflecting on how to implement them in classrooms (as some training programs, inspired by other theoretical frameworks, would recommend), we choose to start from actual practices, inside the classroom and to face directly some aspects of their complexity. From this starting point, the teacher educator tries to make the student-teachers trace back to the generalization of some questions or problems experienced by all teachers (about content or pupils' activity or ways to deal with pupils' activity etc.). The role of the teacher educator is fundamental here in order to allow this movement towards a more general point of view.

4.2. Ways of working with video

From our points of view, teachers' choices in their practices are essentially considered as the result of multiple constraints (complexity). The teachers' decisions are also supposed to be driven by underlying logics for action, at least partially explicable. Therefore, the aim with future teachers is to allow them to be aware of the constraints, the tensions that may appear between preoccupations, choices that are made (including adaptations to contingency) and of other possible choices ("marges de manoeuvre"). In order to complete this objective, teacher education is organized to make teachers develop a reflexive posture on the activity of teaching mathematics, oriented towards didactical concerns. In particular, it includes the ability to manage and evaluate pupils' activity as the result of the teacher's choices (even if pupils' activities, of course, do not depend only on the teacher's choices). It is here that the use of video can play a powerful role in training. Related to this objective, we try to equip teachers with didactic tools which can help them analyze what happens in the classroom and make choices as teachers and evaluate their effects on pupils' learning.

One of the main tools is the *a priori* analysis of pupils' tasks, through the identification of the *adaptations* of pieces of mathematical knowledge (Robert and Hache 2013) inside those tasks. Examples of adaptations of knowledge could be: having to use a basic geometrical relationship within a more complex diagram;

having to use insights about adding fractions to dealing with algebraic fractions; having to move from using Pythagoras' Theorem in 2D to 3D. There is a need to predict what students can currently do and then what extension, or adaptation, of this existing knowledge is needed in the novel context of the task being analyzed. The a priori analysis of mathematical tasks is used both when working with video and more generally, and it is often the first tool that is used when working with teachers on tasks. To analyze mathematical tasks, Robert distinguishes simple and isolated tasks (SIT), defined as tasks where "a single piece of knowledge is used, potentially with simple replacement of general inputs by the given information in the context of the exercise" (Robert and Hache 2013), from tasks where pupils need to adapt the relevant piece of knowledge, "in relation to the required recognitions, initiatives, additions and combinations" (ibid.). Robert developed a list of seven types of adaptations of pieces of knowledge in mathematical tasks. Adaptations are considered by Robert both as means and criteria for learning: being able to adapt a piece of knowledge in a suitable way to solve a task is the sign of a certain level of conceptualization (Vergnaud 1991; Robert and Hache 2013) of it and becoming able to do so is related to the fact of having encountered various tasks in which adaptations of this piece of knowledge were to be made. Moreover, some research results have shown that the way teachers deal with adaptations (choices in scenarios or the way they handle them in classrooms) is variable and these differences have potential effects on pupils' learning (see, for example Chesnais (2013) or Horoks (2013)). The notion of adaptations is one of the tools we intend on offering to pre-service teachers and explains our use of videos. After completing the a priori analysis of a task, we would have students compare it to what happens in the video where a teacher is using this task with his/her students. The a priori analysis allows participants to apprehend the complexity of a mathematical task, and the way the teacher handles this complexity in the classroom.

We also rely on 'The Theory of Didactical Situations' (Brousseau 1997). This theory is "shaped by Piaget's theorization of cognitive development as a process of constructive adaptation and … refined in the light of Bachelard's theorization of knowledge growth as encountering intrinsic obstacles" (Ruthven et al. 2009, p.330). The concept of "situation" refers to the system formed by a problem-solving task and its environment that are especially designed to help the pupils construct some specific new knowledge. We present some of the concepts of this theory to the student teachers, to allow them to analyse and design tasks, but also, when these tasks are implemented in class, to analyse mathematics sessions in terms of phases within a situation (devolution, research, comparison of pupil's procedures, institutionalisation) in a video. Many of these concepts (such as didactical contract, didactical variables, a priori analysis, etc.) are relevant to build situations for the classroom, and to teach or to experiment in class with a research

question. Some of the important elements of this theory are explained in Article 6 and not repeated here.

4.3. Institutional background

Since 2013 in France, future teachers are trained in University structures, called "Ecoles supérieures du professorat et de l'éducation" (ESPE), higher teaching and education schools. The training for primary and secondary school teachers varies with the University where it takes place but always includes, during the second year, both an internship (consisting of taking charge of one or two classes for half the time a tenure teacher usually does) and following courses at the university (in order to validate a master's degree). These courses include ones on didactics and epistemology (about all the subjects for primary school teachers, and about mathematics for secondary school), and aim at helping the student teachers for the classes they have (internship support). Student teachers are also offered general courses about pedagogy (somehow related to the internship) and an initiation to research (in a didactic or educational field) for which they are supposed to produce an essay (a classroom-based action research project, relying on a review of the research literature of the field). Each ESPE is free to decide and organize the content of these courses, so the examples developed below cannot be considered as representative of all ESPEs.

4.4. First example of a session (Chesnais)

I describe in this section a specific teacher training session that I have been implementing for five years in the ESPE of Montpellier in the south of France. Being both a teacher educator and a researcher, the choices made for the internship support course are based on a point of view of teaching practices inspired by my own use of the Double Approach as a researcher. The organization of the internship support course is highly influenced by examples of teacher training sessions and teacher educators' training sessions based on the Double Approach described in Robert and Vivier (2013), Chesné et al. (2009), Chappet-Pariès and Robert (2011).

After a first session (3 hours) where teachers work on the *a priori* analysis of several tasks including several adaptations and discussions about these adaptations, the second session aims at showing them how adaptations might help analyze what happens in a classroom in relation to teachers' decisions, i.e. a means to apprehend the complexity of teaching and learning, in accordance with our main goals.

What is the video and why was it chosen?

The two videos used in this session are part of data collected for my PhD thesis (Chesnais 2009). They show two different teachers in first year of secondary school classrooms (11-12-year-old children). Both videos last for about ten minutes

each and both teachers (T1 and T2) use the same task where students are supposed to construct the mirror image of a given point with respect to a given line, using a set square and a compass.

Activities during the session using video

Before watching the video: student teachers are given the text of the task, on which the students are working in the videos, and are asked to make an *a priori* analysis of the possible activities pupils might develop in response to it. This includes trying to anticipate possible answers and procedures, pieces of knowledge that are necessary to implement these procedures with the potentially needed adaptations, pieces of knowledge that are available at this stage of learning for pupils (in relation to the curriculum). Student teachers are also asked to anticipate how a teacher can orchestrate the implementation of the task in class (organization of pupils' work, material that is needed, timing, interventions of the teacher etc.) and in particular the means that the teacher has to respond to these adaptations and the difficulties that may consequently arise. I focus on the second question of the exercise (figure 1) with the student teachers. They are aware that the method that first year secondary school pupils (age 11-12) could use to complete the task is the one using set square, to draw a line, perpendicular to (AC) through point B, and a compass, to duplicate the distance between B and (AC), and thus find the position of E on the line.

1. Copy the given figure [just the triangle ABC, shown in figure 2].
2. Construct the point E, symmetric to point B with respect to the line (AC).
3. Give, without measuring it, the length of the segment line [AE].

Figure 1: Text of the exercise.

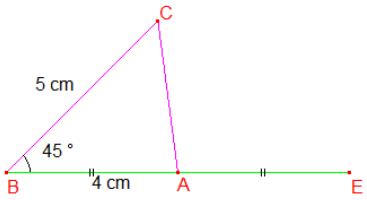

Figure 2. Mistake in the construction, encouraged by the horizontality of segment line [BA].

Nevertheless, the task contains adaptations: one of the main ones is to be able to recognize how to use the method that they know. For example, the line is not drawn, but only the segment line [AC]; there are other elements than just a point

and a line; the horizontal segment line encourages pupils to construct the mirror image of point B on the same line (see figure 2).

During watching the video: after identifying and discussing adaptations, student teachers are shown the videos and are asked to take notes on what pupils and teacher do, or do not do, the pieces of knowledge that appear, and to concentrate especially on what happens about the previously identified adaptations. They are encouraged to try to identify common points and differences between the two videos and especially between the choices made by the two teachers.

After watching the video: student teachers have a couple of minutes to discuss their subjective impressions on, for example, the pupils' levels of concentration, and I then direct the discussion to the choices made about the adaptations. What emerges from discussion is the fact that T1 takes charge of the adaptations (for example by indicating to draw the line before the pupils even start to work on the task); whereas T2 leaves the pupils some time to try to figure out by themselves how to identify the configuration and use their knowledge.

The question then arises of the frequency and time of these kinds of choices in the process of teaching and the possible effects on pupils' learning (about geometry - for example the ability to recognize a given figure in a complex one - and in general), and finally the question about the reasons that may explain these choices (for example the need to control what students are doing in a difficult class…).

Outcomes from the session

The conclusion of the discussion emphasizes the necessity of the *a priori* analysis of the tasks (especially the identification of adaptations that are necessary to complete the task successfully) in order to: (a) choose tasks while understanding what is really at stake in them and organize the teaching of a particular piece of knowledge; (b) anticipate pupils' difficulties and be able to identify them and their origin when they occur; (c) anticipate (different) ways of dealing with them, contemplating possible choices for the teacher.

4.5. Second example of sessions (Horoks)

In the University of Créteil near Paris, I run a course dedicated to the initiation to research, which forms a significant part of students' training (pre-service primary school teachers). The student teachers have to choose from several fields of research to do this initiation, and the following example concerns the course in the research field of maths education. The purpose of this initiation is to give student teachers some objective means to take a step back and reflect upon their own practices. We are also trying to give the student teachers some tools to be able to undertake some actual research (even though they will likely not become researchers in the end), therefore transmitting some selected parts of our theories,

that we consider useful to analyse what happens in a classroom. The student teachers are required to write a fifty-page essay throughout their training, making hypotheses about teaching and learning mathematics, and testing these through an experiment in one of their classes.

We make the hypothesis that we tested in Horoks and Grugeon-Allys (2015), that training teachers through an initiation to research, introducing research tools and methods, might facilitate the development of a more objective stance, in order to reflect on their own practices when teaching mathematics.

What are the videos and why are they chosen?

Using video allows us to look at the practices of another teacher from a researcher's point of view: showing the need for theoretical frameworks to inform how we analyse what happens in the classroom, based upon the analysis of the situation through its mathematical content and the way it is put in use.

The videos that I choose have usually been recorded for research purposes, or within the training program, and usually show a teacher (experienced or not) giving the pupils a task to be solved. As in the previous example, we focus here on a session where we compare two videos that feature pupils in the first year of elementary school (5-6-year-old children), where the task (counting a "big" quantity of objects) leads, in both cases, to grouping the objects by sets of 10, to introduce the decimal structure of whole numbers.

Activities during a session

Before watching the video: I give the task to the student teachers to be analysed first before viewing the video, which raises research questions about the choice of certain didactical variables, and the effects of these choices on the pupils' mathematical activities and learning. In order to inform the a priori analysis, I offer student teachers some ideas about learning the decimal system, inspired by epistemology and research results in the mathematics education field, and let them reflect on the task given to the pupils in each video.

During watching the video: after working on the task and anticipating the students' possible answers and difficulties, students are shown the videos, to analyze the potential gaps between the a priori analysis and what actually happens in the classroom.

After watching the video: the student teachers are invited to comment on the pupils' procedures, when they are visible, compare them to the ones we anticipated, and analyse the role of the teacher in the different phases of the session. The choices of the teacher can then be interpreted, using the Double

Approach, to take into account different constraints that are not directly linked to the pupils' learning. It generally raises more questions, as we usually do not have enough data within the video clip alone to corroborate our hypotheses about learning.

In these two videos, for example, the different choices related to the various phases of the situation (with more or less initiatives for the pupils, more or less time for the pupils' independent exploration and for the comparison of their procedures) might have an influence on the pupils' learning, but it needs to be investigated further, in order to be asserted, in relation to the students' mathematical activity. This is the kind of experiment that could be undertaken by the student teachers for their research essay.

Outcomes from sessions

The goals here are to show researchers' methods - analysing a video as a researcher would do while doing research - and provide the student teachers with necessary tools to begin to interpret teacher practices (and balance the analysis) in a given context (with particular constraints). These are the tools that we hope will help the teachers to reflect on their own practices, but we ask them to take the researcher's posture for now (which can be unsettling for the students) and adopt these methods to enable them to complete the writing of their research assignment. The students are also working with videos sometimes in other parts of their training, but not with a research question in mind (and with or without an a priori analysis of the task or situation, depending on the teacher educator's status, practices and goals).

4.6. Analysis of theory use

Our ways of working with and on videos are driven by the Double Approach and its hypotheses about teachers' practices and their development. The theoretical tools from this theory (to analyze teachers' practices) are both at the origin of our decisions for teacher education (espoused and enacted theory), and at the centre of the video analyses conducted with the student teachers. Therefore, they are relatively explicit in our practices as educators, as intended theoretical and/or professional tools. These are tools that can also be used to analyse our own practices as educators and, explain our choices in a given context.

5. Analysis of use of video in relation to the use of theory

From the French perspective, Julie and Aurélie would say they do not espouse theory on mathematics teacher education, but rather have theories about teaching that inspire hypotheses about working with teachers: Activity Theory informs their way of thinking about how teaching practices work and how they develop (in a comparable manner to Robert and Vivier (2013), Chesné, Pariès and Robert (2009), Chappet-Pariès and Robert (2011), but neither this theory nor the Double

Approach are theories that are meant to be teacher education tools. Their work in teacher education is more a 'logic of action'. For Alf, enactivism, being in part a theory of cognition, can be put to use in thinking about teacher education as well as mathematics teaching. It is from an enactivist perspective that the way of working on video is conceptualized, in which participants begin with a description of the detail of events.

Enactivism is committed to the non-separation of knowing and acting ('all doing is knowing, all knowing is doing', Maturana and Varela 1987, p.27). One of the insights of enactivism is that we live most of our lives in 'readiness for action' (Varela 1999, p.10) responding immediately, and effectively, with those around us. Alf's aim in using video is to support the development of 'readiness for action' in the classroom. The 'espoused' theory of enactivism suggests ways this can be enacted (focusing on the detail of events and initially avoiding evaluation before moving to an interpretation and labelling new distinctions).

From the French perspective, the influence of the TDS can be seen in the way that Aurélie and Julie use problem situations to trigger the identification of needs and questions, and the need for tools (such as categories, established by researchers, to classify mathematical tasks or identify moments of the sessions) to analyze practices and what happens in the classroom, and also the organization of the training sessions into different phases. The influence of TDS can also be seen in the way Julie and Aurélie both start by getting student teachers to do the task that features within the video recording. From a researcher's point of view on the training session, one could mainly use the Double Approach to interpret the teacher educators' choices (a priori or a posteriori) taking into account their professional constraints (type of audience, personal background, research, etc.). This theory prompts the use of two contrasting videos, in both scenarios.

It is apparent, looking across the descriptions of teacher training, that there are differences in relation to the role that theory plays. From the French perspective, in the case of the sessions described in this chapter, not everything comes from the pre-service teachers' practices, Aurélie and Julie bring something new, that comes from their research background: elements of theory, more or less transposed to be used in teacher education, as 'expert didactical tools', which is a different way of building a teacher education program, giving a more or less important role to the students' practices.

For Alf, the only explicit use made of theoretical constructs to guide teacher talk, is the distinction between description and interpretation. This distinction is enforced if needed, by Alf, so that it is enacted in the discussion of video. Aurélie adapts tools from the Double Approach that student teachers use to inform discussion of video and their subsequent planning of activities for the classroom. For Julie, the

work on video entails a deliberate use of the Double Approach and TDS which student teachers use in analysis and, subsequently in writing an assignment.

We can also see the more or less important role of mathematics in these sessions (the videotaped and training sessions) in terms of what is explicit. In the French perspective, the theories used give significant importance to mathematics and to the specificities of the mathematical objects that are studied, which leads to focus also on mathematics during the training sessions. In Alf's description, although the context is mathematics teaching, the way of working is potentially more general. From an enactivist perspective, if teachers are supported to make new (to them) distinctions about what they see on a video, then they are developing theory and hence discussion is about their own theorizing. What is made explicit is this theorizing, and not Alf's espoused theory. We referred, at the start of this article, to a workshop we co-ran at a conference, exemplifying our uses of video. It was apparent from discussion that the detail of the mathematics was more present in the talk during Julie and Aurélie's way of working than Alf's, although such detail would not be precluded from Alf's methods.

There is a difference in the amount of theory that we are trying to get student teachers to engage with and understand, by communicating elements of it, or not, more or less transposed, during sessions. While theory informs Alf's actions, the intention is not for teachers to become committed to enactivism – indeed, as suggested above, it would be unlikely that the term 'enactivism' is used at all during a training session. The intention is to support teachers to develop their own teaching and theorize their own practice. However, the observation/interpretation distinction is important for teachers to use. Aurélie teaches chosen elements of a theory, as tools to teach and analyze teaching practices. Julie teaches theories as tools to conduct some research as a detour to help the student teachers to reflect on their practice. These differences can be related to the goals of the training sessions in each example, and in particular in the last one, where research tools are among the content to be taught to help the student teachers achieve the writing of a research essay. But the fact that we, as educators, make choices about the extent to which theories are visible or not to the student teachers during training raises important issues. We summarize these similarities and differences in table 1.

	Alf	**Aurélie**	**Julie**
Espoused theory (informing session planning)	Enactivism	Activity Theory Double Approach TDS	Activity Theory
Enacted theory (used in training sessions)	Interpretation / observation distinction	Double Approach	TDS and Double Approach
Intended theory	Teachers' own	Tools to analyze	TDS to engage in

(for teachers to use/adopt)	theorizing to support new action	tasks inspired by Activity Theory and Double Approach	classroom research and also to be used as a tool to organize teaching
Role of mathematics in the sessions	Dependent on observations of teachers	A priori analysis of task on video	A priori analysis of task on video

Table 1: The roles of theories and mathematics in our work as educators.

Conclusion

We began with the questions: what guides the planning of video sessions? what guides the action of facilitators during sessions? and, what are the intentions, in terms of teacher learning? The table above summarizes what we found. One of the main similarities in the three examples presented is the way we all start from the actual needs of the student teachers, videos are then an artefact that allows these needs to arise: being close enough from what teachers do in the classroom but with sufficient distance to make them able to reflect on it, especially because they are not directly involved in the situation. They then have access to the complexity but without being responsible for dealing with it (cf. Gaudin and Chaliès 2012). The idea here is close to one developed by Robert when she suggests that there is something like a Zone of Proximal Development for teaching practices: she calls it the Professional ZPD (PZPD): training programmes would allow teachers to take advantage from them if they reach this PZPD. The idea of PZPD is in both perspectives as we all start from student teachers' actual practices and representations about teaching.

Questions remain for us, to research the effects of our choices for training, and especially the place and impact of theory, which is more or less present in our three examples (from making one theoretical distinction explicit (Alf), to making use of a transposition of theory (Aurélie) to inducting to the use of a theoretical framework (Julie)). As stated, our choices can be linked to the differences in our goals and audience when running the sessions described here, but we need to ask ourselves what kind of tools we want to offer to (student) teachers, and what such tools might occasion in relation to their professional development?

We do not consider that the ways we use videos are the only or best ways for teacher education. However, Gaudin and Chaliès (2012) suggest that there is not a lot of reflection in teacher training programmes about the various ways videos are used. The comparison of our practices in mathematics teacher education, around the use of video is helping us to understand our ways of working, as researchers as well as educators, which is in itself a significant step, but also leads us to clarify for ourselves the ways that the theories we use as researchers can influence our work

as educators. We continue to share cultural differences and keep reflecting more about our role, as educators as well as researchers, and the way they both influence each other. The framework of espoused, enacted and intended theories helps us to reflect on our practices and become aware of choices that we may not have questioned, 'expanding the space of the possible' (Davis 2004, p.184) for us as teacher educators.

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