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FRENCH AND ENGLISH THEORETICAL PERSPECTIVES IN MATHEMATICS EDUCATION RESEARCH: AN OVERVIEW AND DISCUSSION OF KEY ISSUES

Abstract. In this article we focus on issues related to theories in mathematics education as used in both French and English settings. As the final article in this special issue, we review the earlier articles and focus on the key ideas and issues which stand out for us. As with the other articles, we seek to address both common and contrasting perspectives, drawing on the examples which illustrate uses of theory. We end by pointing to issues of validation, scale and policy which challenge both groups and look towards facing such challenges jointly.

Keywords. Comparison of theories, mathematics education research, uses of theories, challenges for the research

Résumé. Des recherches en didactique des mathématiques anglaises et françaises : bilan et mise en discussion des perspectives théoriques et des principales questions abordées. Dans ce numéro spécial nous nous sommes centrés sur différentes théories utilisées dans des recherches anglaises et françaises sur l'enseignement et l'apprentissage des mathématiques ainsi que sur les formations des enseignants. Ce dernier texte revient sur les articles précédents, en mettant en perspective les théories et les principales idées et questionnements développés dans les différents exemples abordés. Nous nous attachons à dégager ce qui est commun et ce qui diffère. Nous terminons en revenant sur les problèmes de validations, d'échelles des recherches et de politique, qui constituent des défis partagés par les chercheurs des deux pays, en réfléchissant à des moyens communs d'y faire face.

Mots-clés. Comparaison de théories, recherche en didactique des mathématiques, usages de théories, défis pour la recherche

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Introduction

In concluding this Special Issue, focusing on French and English theoretical perspectives in research in Mathematics Education, our aims are twofold:

- To pick up threads from Article 2, in which we presented key aspects of the two perspectives, and to synthesise similarities, complementarities and differences;
- To reflect on the collection of Articles in the special issue and the richness of theoretical ideas that they bring to the overall picture.

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In our discussion we weave together the substance and issues in these two aims, using Articles 3-6 to provide the rich examples to discuss issues in theory. The discussion is presented partially as a 'conversation' between the two authors, indicated explicitly by a name at the beginning of a paragraph or section. Otherwise the text is joint. The individual paragraphs/sections express a divergence in perspectives whereas, in the joint paragraphs/sections, we are largely in agreement.

1. Dialogue on our joint enterprise in this special issue

Aline: As we saw in Article 2, the development of the two scientific domains, the French and the English (in mathematics education and didactics) was quite different – to summarise their theoretical development with a Bernstein formulation, we can evoke a 'horizontal' development for the English part and a 'vertical' one for the French part. That is to say, for instance, that there were multiplicities of theories adopted in the research in the English case, in common with the other Education Sciences, and only few main ones in the French case, with a more indirect relation with the Education Sciences. In other words, the English development was built *inside* existing theories in Education Sciences, while the French one was built against (in contrast with?) the existing theories in the Education Sciences, and reinforcing the differences. Actually, the *a priori* stage of mathematical analysis (before research into didactics and pedagogy) contributes to the French singular approach and it still characterises almost all of the French research. Such mathematical analyses are less present in beginning stages of the English research, and may even be implicit in it.

Barbara: The idea of the horizontal and vertical makes sense. What seems an important difference is that French researchers throughout France are using the same theories, albeit in response to their own research questions and directions of study. In the English context, different researchers use different theoretical perspectives in relation to their research questions and directions of study and may not even agree on the use of certain theories in specific contexts. This makes for a complex theoretical debate within the English research community. With regard to the French focus on mathematics, conducting a mathematical analysis before embarking on other aspects in a study, I recognise a) that we generally do not do this, but (b) this does not mean that the nature of the mathematics is unimportant. As you have suggested, it is largely left implicit.

Aline: Here we could add something about the researchers in both cases: another difference may arise from the fact that most teacher educators in the UK have been teachers themselves. This is not the case in France, where, at the beginning of the development of didactics, the researchers were mostly university teachers teaching mathematics to undergraduate students with some of them teaching mathematics to

in-service teachers. Perhaps this points to differences in the institutional positions and expected work, and maybe the French researchers had more opportunity to build theories, instead of using existing ones, not directly applicable to teacher training for instance. They have in mind the elaboration of a (missing) general theory for mathematics learning; they did not face directly the needs of applications to schools or for teachers.

Barbara: While it is true in the UK that most teacher educators have been (school-) teachers, there are UK mathematics educators who have not been. This raises the question of who are the people doing research in mathematics education in the UK. The answer is that, many are teacher educators who teach prospective mathematics teachers in the university, but not all. Teacher education is mostly a one year course leading to a PGCE – Post Graduate Certificate of Education. This does not include subject studies. So teacher educators do not usually teach mathematics, per se. Where research into mathematics learning and teaching in higher education is concerned, most researchers have not been teachers in schools, but they are likely to have become university teachers, teaching mathematics to undergraduate students. I certainly think that mathematics educators in the UK are more concerned with analysing teaching and learning, using theories which seem to make sense for their analysis, rather than working explicitly on the critique, development and unification of new theories.

Aline: To go back to the differences, we see that the presentation of the results in Section 3 of Article 2 is different: the English section is organised around five themes relating to what has been learned through research whereas the French section is organised according to what may be enlightened by each theory. But it is interesting to notice here that the English themes are also addressed in France. The main common one would be *meanings in mathematics*, as almost all the French research is concerned with many aspects of mathematical meaning. Equity studies in mathematics exist in France, but they are not as important in France as in the UK. *University studies* were begun from early years (1981) but have not represented an isolated theme until recently, with the development of transition themes, and particularly transition from high school to university. And, in France, there is not really a *policy* theme, apart from the recent studies about the international evaluations. So, behind a real diversity, there is much work on the same issues in the two countries.

Although our theoretical perspectives may seem quite different, we are concerned to study and know more about the same concerns and issues. We address now some particular examples of this.

2. Differing conceptions and practices using the same theoretical perspective – Activity Theory

2.1 Different conceptions of Activity Theory

Article 3 points out differences between researchers in both countries who use Activity Theory (AT): in brief, there are differences both in interpreting and using AT.

Barbara: The English interpretation is broader than the French one, involving a range of general categories in analysis of activity: for example, Engeström's categories of 'division of labour', 'rules' and 'communities', or Roth and Radford's 'contradictions', or Leont'ev's 'motives', 'actions' and 'goals'. These concepts and constructs have been in the public domain for many years, relating back to the origins of Activity Theory; they are well known and discussed. It has made sense to apply them, sometimes with adaptation, to issues in mathematics education, particularly in analyses of classroom interactions and the activity of teachers and students working with mathematics. Teachers and students can be seen as operating within quite different activity systems. Concepts of mediation, goal-oriented action and use of tools and signs allow analysis of complex educational settings, the tensions and contradictions which arise in practice, and the wider contexts and cultures that influence classroom activity.

Aline : In contrast, the French researchers are much more focused on the activity constituted by teachers and students as they work together on mathematics. They have introduced the 'double, ergonomic and didactic approach' to analyse the complexity of activity in classroom interactions. Taking into account this complexity leads them to broaden their analysis. They use cognitive and mediative components to describe the teacher's choices about content and classroom' implementation (where the activities are more visible). But they complete these descriptions including informations on the personal component, to take account of the teacher experience and knowledge, and on the institutional and social components about the corresponding adaptations of the teacher. As part of this same approach, the French researchers have operationalised Vygotsky's model of Zone of Proximal Development (ZPD) for mathematics, looking to bridge the gap between the teacher's *a priori* mathematical expectations and students' mathematical achievements expressed through the concept of 'proximities' in a mathematics lesson.

2.2 An example of different perspectives taken to study what occurs in a classroom

In Article 3, the common issue is a micro-level analysis of a real implementation of teaching, with studies of data from recordings, video (French) or audio (UK) made

in the classes. The research aim is to understand what seems to occur when students hear and interpret the teacher's words and what may motivate the improvisations and comments of the teacher. In what ways do the students benefit, or not, from the discourse? Does their learning improve? How are the teachers' responses moderated towards her/his perceptions of student understanding of the mathematical concepts discussed? The two studies enlighten complementary aspects of these issues: from the English perspective, analysis of the contradictions that are inherent in the episode help to reveal and address issues in communication and their impact on classroom discourse. From the French perspective, analysis reveals the comments some teachers develop to draw students nearer to the teacher's mathematical goals. In the French perspective, the cognitive aspects are mostly taken into account, whereas in the English perspective, more global aspects of the situation become part of the analysis.

2.3 An example showing a possible use of AT as a lens to study a situation of classroom practice with technology

Article 4 discusses the use of technology in the classroom and theory related to this use. Although the theories to be applied in the two cases are different, the teaching-learning outcomes have many factors of similarity. Indeed the authors write:

> 'In some sense, our methods look at two sides of the same coin, teachers' classroom practices with digital technology, from our two different cultural perspectives'

In Article 4, the French case talks about opening up mathematics to student exploration in which the teacher is often in improvisation mode. The teacher has prepared the task carefully with expectations of what students can achieve by using the software as he has set it up. This seems like the equivalent of an *a priori* analysis of the mathematics. However, the student cognitive activity cannot be completely predicted – the expectations are punctured with challenges (tensions/disturbances) arising from the use of software leading to tensions in the planned cognitive route (in that students' activity does not fit with teacher's desired outcomes).

From an English perspective, it seems possible to theorise this through the third generation activity theory triangle of Engeström in which we see tensions between the tools used (the DGE[1] and open task) and the rules and division of labour. The rules of *a priori* analysis leading to tight control of pupil cognitive outcomes are challenged by the open nature of the task and by the DGE imposing its own dynamic in the activity; the expected division of labour, with teacher activity and

student activity separate and well defined, is challenged by the need for teacher and students together to evaluate the reasoning deriving from unexpected DGE outcomes.

In terms of Vygotsky's ZPD, as used in French theory, we see the DGE as an important mediator with both students and teacher gaining new insights through their (joint) addressing of the unexpected geometrical outcomes.

3. Problems arising when researchers have similar aims but different theories

In Article 5, the three authors talk about theory, by means of examples from their practice of working with teachers. As they explain, Coles works from an Enactivist perspective whereas the two French authors, Chesnais and Horoks, use the Double Approach (DA) and seem influenced by Theory of Didactical Situations (TDS). They wanted to choose a video of classroom learning and teaching which all three could use to demonstrate differences in their practices and theoretical perspectives. All three of them believed that choosing a video which all could use was an important task in making clear to each other the very different objectives for each use of the video chosen. However, it was very interesting that they could not find one video that would work for all three.

Barbara: In the proposed UK video, the task was too open for the French – it did not lend itself to clear *a priori* analysis in order to articulate precisely the mathematics that students were supposed to come to know. Whereas a narrowing of the task would be more appropriate for this. I conjecture that, for Coles perhaps, the proposed French task was too narrow for his purposes.

Barbara: From my UK, very practical perspective, in designing classroom activity, we want to present a base for mathematical inquiry in which students can be challenged to think themselves into the problem posed which can be rather broad in scope. The mathematics is thus not narrowly defined. This requires a lot of the teacher since she has to deal with many possible ways in which the students interpret the situation – she has to respond to these in ways helpful to the students (supportive and challenging in varying degrees).

Aline: The pre-analysis of the mathematical task may help the teacher act in the classroom and it allows the researcher and the educator to have clear expectations of the mathematics to be learnt by students. It supports the complexity of teaching decisions and allows the teacher to keep the mathematical discussion focused. In this French perspective, if the task is an introductory one, its *a priori* analysis facilitates the teacher's telling of the knowledge at stake. The teacher is expected then to generalize it apart from the students' use of a contextualized form of the required knowledge on the problem. If it is another task, its *a priori* analysis facilitates the teacher's understanding of the students' precise work with the

required knowledge. It lets her modify the task if it does not fit well enough with her expectations. If it does, it lets her choose her interventions during the students' work, thanks to a deep interpretation and some adapted improvisation, taking into account what occurs, maybe detecting implicit factors as described in Article 3. Actually the teachers are not expected to analyse each task in such a way but it is important to be able to share some of them on important tasks and to enrich their awareness of the particular students' work.

Aline: According to the mathematical content, the tasks choices and their implementation are basics (essential) to let students actually experience in a precise context some of the knowledge to be achieved. Far from a reduction of the students'activity, it may be seen as a whole development process but it requires a lot of the teacher since she has to pick up in the students'work what may worthwhile generalisations or applications.

3.1. Different theoretial perspectives and what they can reveal

The differences in choice and use of theories in Article 5 allow us to reflect further on theory and its use in classroom settings.

Theory of enactivism (Barbara)

Comparing the practice and theory of the three authors of article 5, I think that the three researchers are trying to achieve different outcomes. The English researcher is using an enactivist frame to draw teachers into being enactivist practitioners through his work with them on video. For me, the use of enactivism here can be seen as follows:

Enactivism is sometimes described as 'a path laid while walking'. Students are presented with a very open task. It challenges them to engage and explore possibilities. As they engage, they 'walk'. As they walk they think about the task and start to make some sense mathematically. There may be several different paths for different students. If students discuss and collaborate, these paths can merge or cross, so that the challenge gets modified and the path becomes shared to an extent. We can see the teacher's role as a listener and guide, asking suitable questions, prompting and probing to support and/or challenge students (cf Jaworski's Teaching Triad). Students have to get used to the fact that there is not just one way or indeed one right answer – this is part of enactivism: becoming aware that there are many paths and that it is their own actions that can help them to find a path in fruitful directions. The teacher supports this in different ways. It is very challenging being a teacher within this theoretical frame. Coles uses the enactivist frame to challenge his teachers. They have to see the video and avoid putting their own interpretation on what they see. They cannot 'see' into the minds of teacher or students in the video. They have to limit their responses to the video in terms of

what they can see literally. This forces them to be more aware of the choices a teacher faces and from which she chooses her responses to the students. The teachers observing start to be aware of this multiplicity of choices and perhaps become more aware of the complexity for the teacher and the responses that could be made. Which of these choices best supports or challenges the students is then open for discussion in the group. Their reflections on the video enable them to address their own practice and the choices that they make themselves, enabling them to make more informed, not 'better' choices since it is hard to define what is better. This can be a focus of discussion through which they again develop awareness. These layers of developing awareness form the 'path laid while walking' for these teachers.

Theory of the Double Approach (Aline)

The French researchers in Article 5 are trying to pass on some of the *a priori* analysis tools, built by reasearchers in mathematics education, that seem relevant to reflect on a mathematics session, before it and after it.

The DA does not inform directly students 'activity'. It informs teachers' activity by the way of their relation with students' activity. So these analyses may help to understand the students' activity by a better understanding of the teacher's choices. The teacher training involves a specific approach, based on DA for what concerns practices and on TA and TDS for what concerns the learning, according to the students' grades. For the training, the common idea is to let teachers appropriate some of the tools used for didactical analysis, taking into account that their practices are complex and are not only guided by the students' learning. There are different means to get it but they may be not 'direct'.

For us, it is more important to differentiate between the teacher's point of view and the researcher's one regarding the importance of an *a priori* mathematical analysis inside a whole conceptualising process and as a reference to study videos. A pre-analysis of the mathematical task allows the researcher and helps the educator to have clear expectation of the place of the task in the whole process leading to the mathematics to be achieved. There may be differences between tasks: some tasks facilitate the students' expected work before the teacher telling, some tasks are useful to reinforce the general presented knowledge by exercices, some tasks contribute to have available knowledge, as detailed above.

Some researchers prepare lessons 'ensuring' the knowledge to be achieved, particularly the TDS's researchers for primary level. For instance, they elaborate introductory tasks with a high potential of students' learning, leading to institutionalising the knowledge, provided the teacher's implementation fits the expected goal during the whole process. Using these tasks presupposes the way the teacher is going to intervene: the deal is to let students work by themselves on the

tasks and then to make a bridge (to establish proximities) between what the students know or have done and the general knowledge to be achieved (cf. Article 6 – discussed below). It may be by displaying links (relations) between the contextualized knowledge, as used by students in exercises, and the general knowledge to be achieved, as told by the teacher in the specific moments of teacher telling. It may be before these moments or after, depending on the contents.

But not many teachers use such tasks, not only because it is difficult to implement but also as there are not such studies for each content, particularly in the secondary level, and it is difficult for the indivdual teacher to prepare such a corresponding scenario.

It is then useful to understand what occurs in ordinary classes, using AT theory (as exposed in Article 2), as a reference for analysing students' learning completed by the DA (Article 2) as a reference for analysing teaching practices. In these classes, some researchers study for instance the opportunities to get the students nearer the knowledge to be achieved, whatever the used tasks, and detect the missed occasions, trying to find reasons for them. These reasons may be tied to mathematics, for instance to the choice of the tasks, and/or to their implementations, for instance a student's difficulty may be unrecognised. It may lead to try to develop a kind of teachers' vigilance (care?) on some precise and problematic points, involving the students learning, tied to the tasks, the lessons and what occurs during the class. But these reasons may be also tied to the complexity of what the teacher has to do - managing heterogeneous students, with not enough time, and submitted to various personal, social and institutional constraints. The DA informs the researcher on what has to be taken into account to understand teacher's activity including this complexity.

Finally, the question on teachers' training involves the complexity of practices and some results of research based on the DA. For instance the stability of teachers' implementations, teachers' practice and the importation of the ZPD model for the practices' devlopment, leads us to take into account the teachers' implementations and to lean on the previous teachers' expertise to enrich it.

4. The same theoretical perspective but different situations

In Article 6, we find two different perspectives of using TDS to analyse teaching settings. The first comes from Norway, working within the English domain; the second is from France. In the first case, we see a researcher studying teaching practices in a teacher education setting in which the student teachers are learning mathematics in activity prepared by their mathematics teacher. Here the focus is on the way TDS may be a tool for the researcher to understand teaching practices and to help teacher development. In the example given, although no explicit *a priori* analysis of the mathematical content has been made, there is an understanding of

what this mathematics consists of and of what is expected from the adidactical setting in which activity is rooted. The second case presents a collaboration between teachers, teacher educators and researchers giving them a common aim in designing resources for the teaching of geometry. The analyses with TDS show the way TDS may help the collaboration between researchers-teacher educators and teachers, in research on teacher development.

Barbara: I see in the Article an elaboration of TDS, explaining different aspects of the theory. Key concepts of *milieu*, both didactical and adidactical, and stages of *devolution* and *institutionalisation* were introduced. Although the contexts of the two examples were very different, it is possible to see how this theoretical perspective served an analysis of each of these settings. In this respect, having the different settings and seeing the same concepts related to each of the settings helped to make clear the main elements of TDS. In some ways, I see a value in the key concepts mentioned above for any setting in which a teacher wishes her students to learn specific mathematical knowledge. A difficulty arises when the tasks (didactical or adidactical) are predesigned by researchers or teacher educators with the expectation that a teacher can fulfil the designed teaching approach without having been a part of the original design/planning. In the first example, we see that the teacher is part of the design process, and in the second example, there is collaboration between teachers, teacher educators and researchers in the design. Thus this difficulty is avoided.

Aline: The framework TDS is particularly concerned with the design of learning situations, and also to analyse what happens in class during the progress of the actual implementation of the situation, in reference to the design, and, more recently, to identify questions useful to develop teachers' practices. But the main aim remains to study the cognitive potential of a given situation, that is the study of what the students may learn according to the contents' choices, mainly the tasks and their implementation, often to introduce a new notion. In both cases of Article 6, as the authors say, 'the focus was on the design of the situation itself and its study'. There are differences, however, in the objectives and research questions in the two contexts, which are training contexts. In each case, the teachers have to learn to use the chosen tasks, adapting them to their students but trying not to lose their potential. The use of concepts of TDS is more explicit for the teacher in the case of multiplication (first author) than in the case of geometry (others authors). In the case of geometry, there is a big difference between the small group (with researchers, educators and teachers) and the large group (of teachers): in the small group, gradually, there is a certain familiarisation, at least a use 'in action' of the concepts of TDS, without expressing them, in the exchanges during the design of the situations and the analyses of class observations; in the large group the focus remains on decisions focused on practice.

5. The use and influences of research results in the two domains

Although the development of the French and English scientific domains have taken different forms (Article 2) with the use different perspectives (cf. Articles 3, 4 & 5), or with the same perspective (Article 6), researchers in both contexts nevertheless tackle some common issues, and the kinds of outcomes we get are not so very different on a gobal scale (the two faces of the same coin, at a more or less general level). Both sets of researchers are concerned to develop the learning and teaching of mathematics, both in theory and in practice. What can be learnt from the joint enterprise, in these Articles, enriches the overall perspectives and emphasises the joint enterprise. However, these findings are not taken into account by the 'decision makers', either in France, or in the UK.

What is interesting is that this occurs in both countries, independently of the development of the domains. It is not (only) the proliferation of the theories that may explain this unwillingness of the institution and policy-makers to seek the advice of researchers or to pay attention to research findings. It is an important result of this common work: there have to be new ways to have some influence; perhaps there need to be international common results to make perspectives more visible to leaders in educational policy. This might, for example, follow the European synthesis of didactical results with the 'solid findings' such as those published by the European Mathematical Society ("Solid Findings in Mathematics Education", EMS Newsletter, September 2011).

However, in the domains in this special issue, the production of evidence (validation?) is not simple since most of the studies are qualitative ones and small scale. More generally, it is clear that there are no obvious means for 'assessing' such studies with quantitative evidence – as is confirmed in the Articles 3-6. Even though some international assessments inform on the state of students' knowledge, it is not directly possible for these to be turned into teaching changes. The relations between quantitative assessments and individual practices are not simple, there is often a lack of adjustment of the exercises to the corresponding teaching, and learning is a long process not reducible to a state that can be measured with a snapshot. It is well-known and concerns almost all Human and Social Sciences but it plays a role in the institutional reluctance. In these sciences, 'robustness' does not come from assessments.

However, in the case of teacher education, where the teacher educators are also researchers in mathematics education, there is growth of awareness of the outcomes and issues from research as researchers communicate both within and across national boundaries. The communication that takes place at national and international research conferences feeds into the professional knowledge base from

where it is distilled by teacher educators in preparation for their work with teachers. It is possible to see this research knowledge permeating thinking and practice through teacher education opportunities. Teaching, as it can be seen in schools today, is influenced not only by policy decisions but also by the teaching of teacher educators, informed by their research knowledge. The 'solid findings' mentioned above can be an important contributor to this knowledge and we need to build this into our research and professional practice.

6. Relation of results of research to the contexts and focuses of the particular studies

We see another factor which weakens research results. The fact is that our results (outcomes) depend mostly on the contexts of the studies and their possible uses depend on situated learning, in a country or between countries. It is very clear in Article 6, where the adopted theory is exactly the same (TDS) but the institutional contexts and focuses of research are different – in one case the researcher studies real pre-service teachers training, and in the other case the researcher studies resources for in-service teachers' training. In the first case analysis reveals differences in the conceptualisation of tasks by the teacher and the mathematical activity of the students in working on these tasks. These have implications for the design of tasks more generally and for the work of the local practitioners more particularly. In contrast in the second case the issue is to find a resource available for many teachers. It leads to a first common analysis of the mathematics involved but then to a different analysis of the discussion on the variables and the way of presenting the research. This great dependence on the contexts may explain some lack of our influence, tied to the complexity of the way of adapting results to many factors. Programmes differ from one country to another, cultural habits too and even inside a country, teachers may develop some different ways of teaching to be comfortable in their craft; students are very different according to their family for instance, but not only. So one result has to be presented in the context it was obtained, with its limits and without obvious more general impact.

Another reason for a collective lack of influence is that local, qualitative analyses are more frequent than global ones. The shift from local studies to their global interpretation or use is difficult, precisely because of differences in context to the time they take and to the large amount of data to be gathered. Then it is hard to take into account all the variable parameters involved.

It is then difficult to infer global results from our local analysis for the students' learning. Researchers have only hypotheses on the quality of the scenarios. They suggest that the recurrence of teaching ways is an important factor for students' learning. An example can be seen in Article 3: in the French classroom episode, an *a priori* analysis of the mathematics in focus is done before activity takes place in

the classroom, and informs the analysis of this activity. In the English episode, while mathematics learning and teaching is central to analyses, the mathematics itself is largely implicit in the analytical treatment of the episode.

A final reason for the lack of use of the research cited in this special issue, tied to the previous developments, may come from the fact that direct 'ways of doing' are not the aim but, rather, the aim is towards tools to understand what occurs and to elaborate and adapt the teaching as the lesson progresses, according to what occurs in the class with the students (cf. Article 5). It has to be somehow different from content to content, from one day to another, from one class to another and from one teacher to another. The results are not spectacular, there are no simple statements to pass on, they involve complexity which is not easy to communicate. While it is very important to understand what may be common in our works, not only for researchers to understand each other, but also for our readers, and specially the non-specialist ones. We see that Articles 3 to 6 allow us claim that there are many common issues addressed by the research, and that, in spite of differences in goals, long-term intentions, theories, methodologies, unit of analysis, data and contexts of studies to tackle these issues, the results may be considered as two aspects of the same reality. To say it in other words, it is possible to include these results in a one 'bigger' result. This may perhaps contribute to a better visibility outside the field of mathematics education.

However, one thing to observe is that we see cross-national studies in the EU which seem able to deal with a range of contexts, cultures and data collection, often with shared data and perspectives for analysis. A difference with what we are discussing above, these studies are conceived in advance, the theoretical perspectives are stated and agreed up front, as are methodologies and shared practices. These pre-arranged commonalities enable cross-national comparisons and wider impacting outcomes. One possibility from the insights that the joint activity for this special issue has revealed is for further joint research, although sources of funding are hard to acquire.

Conclusion

To conclude this article and the whole Special Issue, we have to describe some 'benefits' of this common work and open some perspectives. It is clear that the deepened discussion between researchers of different countries contributes to a deeper understanding of each point of view: we not only learn about each other's perspectives but we get new insights into our own perspectives. On the one hand, the discussion on the same themes, with the precise work on examples, was really very productive to let us enter the others' overall approaches and motivations. The contrasting of our micro-level analyses has been relevant to make us think about the issues, the methodologies and the results. Indeed to make others understand our

work more exactly contributes to making us explain more deeply some elements we may never have made explicit and even to detect implicit chacteristics in our approaches which benefit from being made explicit.

On the other hand, one perspective may be to present in a single (simplified) form our various results – as two faces of the same regularity. For instance for teachers training, the main result is perhaps the necessity that all the researchers claim, of making the teachers become conscious of the students' needs, of the necessity of listening to them, and of giving them effective tools for their learning. It is also becoming clear (as evidenced in both English and French cases) that the collective study of videos may contribute to our main goals – whatever may be the way to reach this consciousness. The contrasting of the methods and of the fine results is perhaps less interesting for the rest of the world.

This unified presentation of our results may be easier in such a common work, in a second phase after the first phase of eliciting the contrasted approaches, and it may contribute to our visibility.

References

All the references in this article are within the references of Article 2 and Article 6, in this volume; the authors chose not to add references is this synthetic article.

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