DYANA WIJAYANTI

ANALYSING TEXTBOOK TREATMENT OF SIMILARITY IN PLANE GEOMETRY

Abstract. This study presents a specific analysis of the treatment of the topic "similarity" by several college textbooks, in this case six Indonesian textbooks. The choice of this mathematical topic is justified by the fact that it requires in the activity of the pupils an implementation of both the geometric register and a numerical register, that of ratios and proportions. Thus, it is appropriate for the development of a general validity reference model, which will require further studies. The two points of theoretical support in the present study were the epistemology of the mathematical topic under consideration and the notion of praxeology, pertaining to the anthropological theory of the didactic. In addition to the meticulous review of textbook contents, the sketched model allowed us to evaluate the proximity of similarity treatments in a textbook and in the formal assessments of students at the end of college.

Keywords. Textbooks analysis, epistemological reference model, similar figures, proportionality.

Résumé. Analyse du traitement de la similitude dans les manuels. Cet article présente une analyse spécifique du traitement de la similitude par plusieurs manuels du collège, dans le cas présent six manuels indonésiens. Le choix de ce thème mathématique est motivé par le fait qu'il exige de l'activité des élèves une mise en œuvre à la fois du registre géométrique et d'un registre numérique, celui de la proportionnalité. Ainsi, il est approprié à l'élaboration d'un modèle de référence de validité générale, lequel supposera des études complémentaires. Les deux points d'appuis théoriques dans la présente étude ont été l'épistémologie du sujet mathématique considéré et la notion de praxéologie, relevant de la théorie anthropologique du didactique. En plus de l'examen méticuleux des activités proposées par les manuels, le modèle ébauché a permis d'évaluer la proximité entre les traitements de la similitude dans un manuel et dans les évaluations officielles des élèves à la fin du collège.

Mots-clés. Analyse de manuels, modèle praxéologique de référence, figures semblables, proportionnalité.

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1. Introduction

Mathematics textbooks do not only present *knowledge* or "general facts", but are also important tools for teachers to engage students in mathematical activities or *practice*; mathematics textbooks always include worked examples and exercises. They are an essential component for the teachers' daily work with the textbook and so, in principle, for their choice of using it. To provide a useful and complete analysis of a mathematics textbook, the consideration of examples and exercises is therefore crucial. The precise analysis of practices is especially relevant when the textbook is used in contexts preparing for a centralized, high stakes examination. In such contexts, it is common that teachers concentrate students' work on types of tasks which appear in the examination. One can then ask: to what extent does the textbook present a similar concentration? Does it explicitly emphasize examples and exercises which are closely related to the examination? To what extent does it enable student work on types of tasks which are not frequently appearing at the examination? To choose a textbook, teachers may rely on rather informal answers to these questions; a more precise tool is presented in this paper, for a specific topic (similarity in plane geometry).

At the same time, more global features and connections within the textbook are important and should be related to the more local analysis of the practices which the textbook proposes to students. For instance, a central result on similarity of polygons concern the proportional relationship of corresponding sides; this could be explained with more or less explicit mention of the relation to the work done on ratio and proportion in arithmetic (usually in earlier grade). Explicit connections between different mathematical domains (here, geometry and arithmetic) are considered important to teach students that mathematics is a connected body of knowledge, and to avoid the tendency of "thematic autism" – that is, mathematics teaching that makes students visit one theme after the other, with no connection between them (Chevallard, 2006, 2015). We can make such a connection with the analysis of proportion in arithmetic in textbooks, already presented by Wijayanti and Winsløw (2015). As noted in that paper, textbooks at lower secondary level do not present much explicit theory; still, relations between the two domains may be identified both at the level of practice (e.g., exercises and techniques to solve them, cf. Wijayanti, 2015) and at the level of more informal, expository discourse in the textbook. In this paper, we investigate the relations between similarity and proportion, as presented in the textbooks at both levels.

Concretely, in this study, we construct a "sketch" of reference model to analyse the treatment of similarity in Indonesian lower secondary school textbook. This reference model is justified solely on the basis of a review of some textbooks in a topic. Thus, what we present in this study is not a general validity of reference model which will require a further study. However, we believe that our 'sketch' of reference model can be used to analyse all the theory and practice of textbooks we chose. For further discussion, we will use a "sketch" of reference model as reference model. We show that this reference model can also be used to consider the extent to which the practice on similarity, proposed by the textbook, aligns with the items on similarity which appear at the national examinations of Indonesian lower secondary school. Finally, we discuss explicit and implicit relations, established in the textbooks, between similarity of plane figures on the one hand, and ratio and proportion on the other. In Indonesia, the last topic is treated in 7th grade and the first topic in 9th grade. So, we look for these relations in the 9th grade textbooks' chapters on geometry.

This paper consists of seven sections. First, we present the necessary literature background on textbook analysis, especially studies of texts on geometry. Then, we also provide a short introduction to the anthropological theory of the didactic (ATD) as an approach in textbook analysis, and formulate our precise research questions. In section 3, we explain the context and methodology of the study. In section 4, we present the reference model on similarity, as a main result of this study. Then, we apply the model to produce a quantitative comparison of types of tasks in the textbooks analysed, and the Indonesian national examination (section 5). In section 6, we analyse the connections in the textbooks between similarity and proportion. In the final section, we present conclusive remarks related to the research questions, as well as perspectives for other applications of reference models of the type produced in this paper.

2. Background from Textbooks Analysis

Recently, the use of praxeological reference model has appeared as a new method to analyse mathematics textbook. For example, González-Martín, Giraldo and Souto (2013) developed such a model to study the treatment of the real number system in Brazilian textbooks. A more fine-grained analysis can be found in Hersant (2005), to study techniques for certain "missing value tasks" related to ratio and proportion, as they appear in textbooks for France lower secondary school. Hersant's reference model was developed by Wijayanti and Winsløw (2015) who focused on broader range of proportion tasks in the domain of arithmetic. As a result, they developed a larger reference model, consisting of seven types of tasks that can be applied to analyse the whole theme of "proportion" in Indonesian lower secondary textbooks (where the word theme is used in the sense of Chevallard, 2002b). They also noticed that some types of tasks are common and numerous in all the analysed textbooks, while others are rarer and appear just in a few of the textbooks. In this paper, we examine how this method can be used for a related theme (similarity) in a different domain (geometry) and to what extent the textbooks analysed provide explicit links between the two themes (proportion and similarity).

Miyakawa (2012) presented a textbook study in the domain of geometry, which is also to some extent based on ATD. This study focuses on how lower secondary textbooks in Japan and France exhibit specific differences in the meaning, form and function of proofs in geometry. It is interesting that clear differences may appear between textbooks from different countries and be explained in terms of wider differences between them. A similar comparative study was done by Jones and Fujita (2013), on proof between British and Japanese textbooks.

Sears and Chávez (2014) studied the factors which seem to influence teachers' use of textbooks for geometry teaching in the US, for instance, what make them assign or not assign a given exercise as homework for students. Among the influential factors we find the teachers' perception of

- their students' competences for the given task, and

- the importance of the given exercise to prepare for high stakes assessments.

For instance, tasks involving proofs are rarely assigned because they are considered too difficult for most students and they are considered less important for external assessment purposes. This observation is related to one point in the present paper, namely the closeness of tasks in textbooks to tasks appearing in high stakes examination.

The often-cited risk of school mathematics being disconnected (visit of different "monuments", cf. Chevallard, 2012) encourages many authors to investigate ways to strengthen the relations between different domains of school mathematics. For instance, García (2005) investigated the possibilities to establish relations between two themes in lower secondary school in Spain, namely ratio and proportion in the domain of arithmetic and linearity on the domain of algebra. In the domain of arithmetic, proportion involves ideas like ratio and scale, which are also relevant for the context of linear functions. For example, students are given a proportion task for four relationship numbers, e.g. a_1, b_1, a_2, b_2 so that $\frac{a_1}{b_1} = \frac{a_2}{b_2}$. Then, students are asked to find *c*. Students can solve such task by using "cross product technique": $a_1 \times b_2 = b_1 \times a_2$. This technique can also be expressed based on linear functions $f(a_n) = b_1 . a_n$. For example, students are given 2 kg orange for Rp[1]. 20.000, then students are asked to find the price for 4 kg orange. By using relationship numbers $\frac{2}{20.000} = \frac{4}{d}$, students can find 4 kg orange (d) which is $2 \times d = 4 \times 20.000 \Leftrightarrow d = 40.000$. Additionally, students also can solve this problem by finding the price of 1 kg orange which is Rp. 10.000. Then, students can apply linear function formula f(4)=10.000×4⇔f(4)=40.000. However, García found that there is a poor connection between the two domains in textbooks for this level. This research inspired another point in the present paper, namely the use of praxeological reference models to investigate explicit or implicit connections in textbooks which go across domains.

[1] The official currency of Indonesia is the rupiah (Rp).

At the end of this paper we consider, specifically, how textbooks relate the theme of proportion (in arithmetic, calculations involving scale and ratio) with the theme of similarity (in geometry). We consider both the extent to which explicit relations appear, and also the extent to which the geometrical theme of similarity goes beyond simply arithmetic computations. As pointed out by Cox (2013), many school exercises on similarity of geometrical figures can be done by doing routine computations with given dimensions, without much consideration of the geometrical configurations at hand. She stresses that the tasks given to students play an important role for the extent to which they develop a "truly geometrical" knowledge of similarity and investigates the use of tasks designed with this in mind. Cox's study further motivated our systematic study of the types of tasks on similarity that appear in (a selection of) mathematics textbooks.

Theoretical Framework and Research Questions

Our approach to textbook analysis is founded on the notion of didactic transposition from ATD, the anthropological theory of the didactic (Barbé, Bosch, Espinoza & Gascón, 2005; Chevallard, 1985, 1988; Chevallard & Bosch, 2014; Winsløw, 2011). This notion explains the exchange of knowledge between three institutional levels: the scholarly communities of mathematics (developing and maintaining scholarly mathematics), the level of "official school mathematics" (like ministerial committees, textbook publishers), and school mathematics in ordinary classrooms. We consider only the first two levels (and thus what is normally called external didactic transposition), but use a slightly more detailed model of "official school mathematics", as we distinguish three components that contribute to define it: the official curriculum, given some form of law or decree; the way school mathematics is presented in textbooks; and the official rules and practices for summative assessment (in our context, we have a national written examination in mathematics), see Figure 1. In the figure, we mean to indicate that textbooks are in principal designed to "deliver" the official curriculum, and the national examination is designed to assess the students' achievement of the curricular goals. At the same time, we want to study (in fact measure) the extent to which the exercise material in textbooks align with the assessment practices (concretely, actual exercises appearing in the national examination), suggesting a more direct relation between official assessment practices and textbook design.

To analyse the transposition of specific knowledge and practice between these different institutional levels, we need an epistemological reference model, i.e. an explicit and independent model of the knowledge at stake, to avoid taking the viewpoint of some particular level and also to make our analysis completely explicit. We present such a model for the basic (practice oriented) components of similarity in section 4.

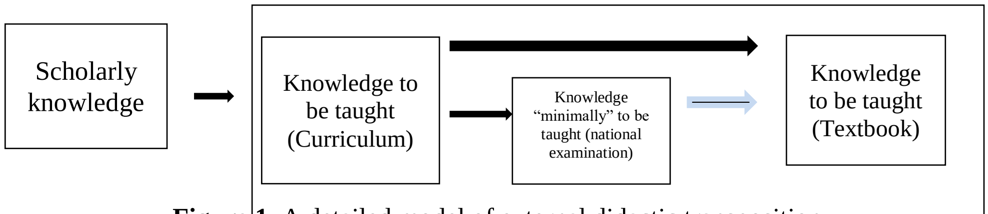


Figure 1. A detailed model of external didactic transposition

We use the notion of praxeology (Chevallard, 1999, 2002a; Chevallard & Sensevy, 2014; Winsløw, 2011) to describe mathematical practice and knowledge as our epistemological reference model. More details on this are available in the references just given, so we simply recall that a praxeology consists of four components: 1. a type of task (T); 2. a technique (τ); 3. a technology (θ); 4. a theory (Θ); thus a praxeology is a four-tuples (T/τ/θ/Θ), and be considered as a pair with two elements: praxis (T/τ), the practical block; and logos (θ/Θ), the theory block. We will use this notion to analyse the practice (examples, exercises) and the theory concerning similarity, as they appear in Indonesian lower secondary school textbooks.

Mathematical praxeologies in school do not only appear as isolated 4-tuples of the above kind. They are connected at different levels which ATD describe as discipline, domain, sector, theme, and subject (cf. the references above and Table 1). Here, the discipline is mathematics (as school subject), it consists of various areas of practice and knowledge like geometry and algebra (the domains); both disciplines and domains are, naturally, determined by the institution, although mathematics and its domain appear in very similar forms at schools across the world. The remaining levels are determined by praxeological levels: a sector is a collection of praxeologies unified by a common theory, etc (see Table 1).

Discipline: Mathematics			
Domain		Arithmetic	Geometry
Sector	Theory	Proportions	Plane Geometry
Theme	Technology	Direct proportion: Ratio and scale	Similarity of polygons
Subject	Technique	$r=x_2/x_1$	$\alpha = dist(Mx,My)/dist(x,y)$
	Type of task	Given x_1 and x_2, find r so that (x_1, x_2) and $(1, r)$ are in proportion “$(x_1, x_2) \sim (1, r)$”.	Given two points and their images, find scale of magnification α.

Table 1. Two related examples of sub disciplinary levels.

In this study, the relation between proportion and similarity goes across domains, as it concerns themes from the arithmetic domain and the geometry domain. It may, for this reason, be harder to establish than two themes within the same domain. In general, there is a tendency that theme, even within the same domain, remain relatively isolated and unconnected in school mathematics Chevallard (2015). Connections across domains could be even rarer, even if they are natural from a scholarly point of view. With this background, we can now formulate our research questions for this study:

RQ1. What reference model can describe the theme of similarity in textbooks?

RQ2. What can be said based on the reference model, on the extent to which the similarity theme in textbooks aligns with the national examination in Indonesia? (Here, types of tasks are the main unit of analysis).

RQ3. What connections between the themes of similarity (in geometry) and proportion (in arithmetic and geometry) are explicitly established in the textbooks?

3. Context and Methodology

Formal education in Indonesia is divided into three levels: primary school (grade 1-6), lower secondary school (grade 7-9), and upper secondary school (grade10-12). Students normally start in primary school at age 7. Mathematics is one of four subjects (with Indonesian Language, English, and Social/science subject) that Indonesian students have to pass with certain minimum marks at the final national examination of lower secondary school. This makes some schools offer additional hours for these four subjects, but also the regular teaching could be expected to focus rather much on the final exams towards the end of lower secondary school. We examine this hypothesis concretely by comparing the types of tasks appearing in textbooks with those appearing in the national exam.

The Indonesian government provides an online resource for teachers and students (the website of BSE: http://bse.kemdikbud.go.id/). On this website, one can download certain textbooks which have been authorized by the government for free. There are also textbooks published by private companies which are authorized for use in schools. But as these are not free as the texts on the BSE website, so the online textbooks are widely used.

In this study, we analyse relevant parts of the online textbooks for mathematics in grade 7 (a total of three textbook: Nuharini and Wahyuni (2008), Wagiyo, Surati and Supradiarini (2008), and Wintarti et al. (2008)) and grade 9 (a total of six textbooks: Wagiyo, Mulyono and Susanto (2008), Agus (2007), Dris and Tasari (2011), Marsigit, Susanti, Mahmudi and Dhoruri (2011), Masduki and Budi Utomo (2007), Djumanta and Susanti (2008). To answer RQ2, we also analyse the national examination tasks in mathematics for the final exam in lower secondary school, in the period 2007-2015.

The first step in this textbook analysis is to identify chapters that treat similarity theme (typically, there is a chapter on "congruence and similarity"). Then, we analyse all textbook's discussion, examples and exercises on this theme. Concretely, we first review discussion part and analyse it epistemologically to find technology and theory of similarity (see section 4.1). Then, we consider examples and elaborate it using epistemological analysis to identify types of tasks and corresponding techniques which they present (see section 4.2), and then identify types of tasks found in the exercise sections (assuming that techniques presented in examples are to be used when possible). We simply classify examples and exercises in types of task: whenever we meet a new one, the model is extended with that type of task. Thus, the model is fitted to the data (here, examples and exercises).

The resulting analysis gives a kind of profile of textbook, as one can expect that much of students' practice in actual teaching will be built around these types of tasks and techniques, along with elements of technology and theory provided in the main text of textbooks. Naturally, at this school level, theory is often "informal", so that even key notions like similarity may not be given a completely explicit definition. Even so, we try to describe technology and theory that is located in the reference epistemological model section. Additionally, the resulting analysis can be used to capture the national examination task related to similarity theme.

Pertaining to connection analysis, we firstly analyse a potential connection by observing the commonality of praxis level in both proportionality and similarity theme. Then, we consider an 'explicit' theory level by noticing the assertion of a 'closeness' of the two sectors and the use of 'proportionality' term in similarity. In the 'explicit' praxis level, we examine the direct explanation of an arithmetic technique in similarity. In this case, we focus more on similarity theme in 9^{th} grade because it is natural to recall knowledge from lower grade in higher grade.

Finally, we use the categorization of types of tasks to make a quantitative analysis of "mathematical praxis" proposed by different textbooks (to investigate RQ1), and mathematical praxis required by the national examination (to investigate RQ2). Finally, we analyse the explicit connections which the analysed textbook chapters point out between similarity and proportion (to investigate RQ3).

4. Reference Epistemological model for similarity

4.1. Elements of the theory of similarity

In scholarly mathematics, similarity may be defined in different theories and levels of generality. Concerning similarity of any two subsets in the plane, one precise definition could be given based on transformations with specific properties. In school mathematics, however, similarity is usually defined more directly in terms of the figures and their properties, and only for special cases like triangles and other polygons.

Here is a very common definition of similarity of polygons in secondary and even college level textbook s (Alexander & Koeberlein, 2014, p. 218): Two polygons are similar if and only if the two following conditions S1 and S2 are satisfied.

S1. All pairs of corresponding angles are congruent
S2. All pairs of corresponding sides are proportional

For triangles, S1 and S2 are equivalent, and so similarity in this special case could be defined using any of them. One can also treat similarity of polygons in general through the special case of triangles, by dividing the given polygons into triangles ("triangulation"), and then similarity of the polygons depends on whether they can be triangulated into similar "systems of triangles" (a very difficult notion to make precise). We do not go further into this as triangulation techniques do not appear in the textbooks (See Euclid's elements of geometry in Fitzpatrick, 2008, p. 176). On the other hand, we will notice what relations the textbooks explicate between triangle similarity and the general case, and in what order they are presented.

A main challenge with the above definition is the meaning of "corresponding"; it could be formalized in terms of existence of a bijection between the sides in the two polygons, satisfying that neighbouring sides are mapped to neighbouring sides etc.; it is an interesting task for undergraduate students to develop the details, which can also be found with various degrees of formalization in some textbooks (Lee, 2013, p. 213).

However, from a didactic point of view, such formalizations may not be relevant in lower secondary school. Here, the notion of "corresponding" will often appear as transparent, for instance when it is obvious from given figures how sides and angles in two polygons correspond to each other. But, this apparent transparency is somewhat problematic: the "obvious" correspondence *requires* that the polygons are similar (so that we can pair congruent angles, and then the sides); but if similarity is to be checked by deciding if the S1 and S2 are satisfied, there is evidently a vicious circle (we can only use the conditions if the answer is yes).

In lower secondary school, even more informal definitions are common, involving visual ideas like "same shape" and carried by a large range of examples. If a semi-formal definition involving "corresponding" sides or angles is given, examples are also likely to be the only explanation of "corresponding". The problem is that in these examples, one never sees "corresponding" angles that are not congruent, or "corresponding" sides that are not in equal ratio.

Corresponding angles can be described by ordering angles in both polygons and observe if the same angles appear. Then, corresponding sides can be paired by applying those sides that located between corresponding angles. Then, students can observe if the corresponding sides have equal ratios.

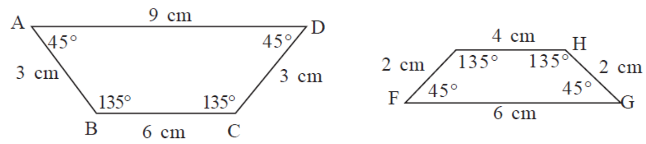
As we describe in the data analysis section, we analyse six Indonesian lower secondary textbooks. All textbooks give definition of similarity by using two components S1 and S2. To explain definition, the authors of five textbooks start with example of similarity in daily life situation, e.g. augmented photograph. Then, students are given two polygons and are asked to observe and/or to measure angles and sides in two given polygons. Then, students lead to the similarity definition. First, we can notice that students are taken for granted to understand the word ‘corresponding’. Second, students miss an opportunity to think about the meaning of similarity by themselves.

We also noticed that all of textbooks treat similarity into two parts; polygon similarity and triangle similarity. Furthermore, polygon similarity always appears first. To connect similar polygon and similar triangle, the authors remain students about definition of polygon similarity: S1 and S2. For example, Dris and Tasari (2011) wrote that “Triangle is also a polygon, and then the definition of polygon similarity is also valid for triangle similarity”. The authors also point out that similar triangle is a special case, because they only need to apply either S1 or S2. Additionally, similarity of triangles can be proved by finding two proportional pairs of sides and two equal angles (S3).

We now proceed to build the reference model for types of tasks occurring the textbooks and at the national examination. The construction of types of tasks are based on epistemological analysis, as explained in the methodology section.

4.2. Practice blocks related to polygon similarity as it commonly appears in Indonesian textbooks

We found four types of tasks in the textbooks, concerning polygon similarity. The description of each type of task is followed by the technique, and an example that is translated in English. Furthermore, there will be a discussion after each example is presented. Type of task 1 (T_1) is to decide if two given polygons are similar (Table 2). In the discussion we will also present some variations of T_1.



For the pair of figures above, determine if they are similar or not?
(Masduki & Budi Utomo, 2007, p. 7)

Table 2. Task related to T_1.

This type of tasks may be generalised, with some caveats:

T_1: given two figures of polygons P and Q, with given side lengths and given angles, determine if the two polygons are similar.

τ_1: order the angles in P and Q from small to large (visual inspection), and see if the same angles appear. If so, verify S2 while regarding "sides between corresponding angles" as corresponding sides.

There are many variations of this type of task. "Variations"/special cases (e.g. where the "geometry" makes some of the definition superfluous). Some examples are given in Table 3.

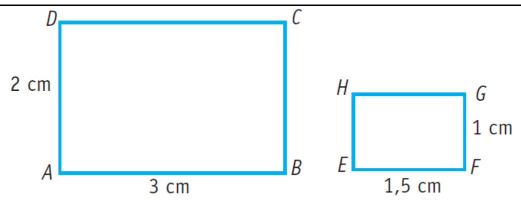
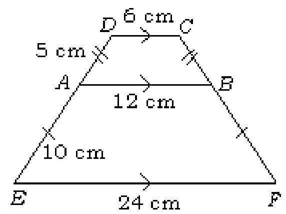
 Determine if rectangle ABCD and rectangle EFGH above are similar or not? (Marsigit, Susanti, Mahmudi, & Dhoruri, 2011, p. 28)	 Are ABCD and ABFE in the picture similar? (Wagiyo, Mulyono, Susanto, 2008, p. 4)

Table 3. Variations of T_1.

In the first task in Table 3, students are given two rectangles, with measures of the sides. In this case, it is trivially true that all angles are congruent. To decide on S2, students can order the side lengths in (smaller, larger) for the two rectangles and compute the ratios of smaller sides and of larger sides. They are equal, the rectangles are similar. While students determine congruent angles, using properties of rectangle (all angles are right angles) in the first task, students in the second task can determine congruent angles, using similar symbols that are given in each side.

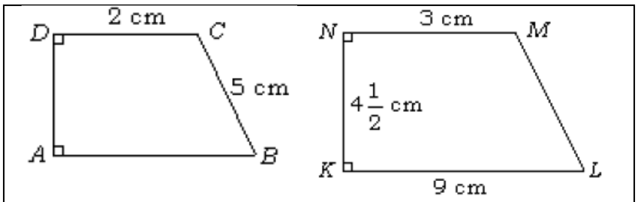
Tasks on two triangles with only angles or only side lengths given, do not belong to T_1. Thus, almost all tasks concerning similarity of triangles will not be of type T_1. In fact, the textbooks we consider all treat triangles apart from “general” polygons, which then have at least four sides.

Different from T_1 in which students are asked to decide on similarity, students in T_2 are given similar polygons with given angles. Then, they are asked to identify equal angles and corresponding sides. We can see the formulation of T_2 bellow.

T_2: given two similar polygons P and Q with given angles and given sides. Identify what angles and sides correspond to each other.

τ_2: identify the corresponding angles by ordering them. Identify the corresponding sides as sides between corresponding angles.

In fact, the technique for T_2 given in most textbooks is less precise or explicit. Mostly, the authors just give the answer without any explanation to solve the task. In this type of task, we will also include triangles under polygons because the same technique applies to find equal angles and proportional sides. An example of T_2 can be found in Table 4.



Observe trapezoid ABCD and trapezoid KLMN.
Given ABCD and KLMN are similar.
a) Determine a pair of equal angles
b) Determine a pair of proportional sides
(Wagiyo, Mulyono, Susanto, 2008, p. 8)

Table 4. Example of T_2.

Often, T_2 is followed by T_3 (see below) where students need some numbers to do calculation. T_2 is solved based on a visual representation. To solve T_3, students not only need a visual representation, but also a calculation technique (Table 5).

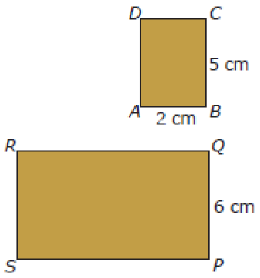
	If rectangle ABCD is similar to rectangle PQRS, determine the length of QR. answer: One condition of two similar figures is to have proportional corresponding sides. So, $\frac{AB}{PQ} = \frac{BC}{QR}, \frac{2}{6} = \frac{5}{QR}, 2QR = 30, QR = 15$ Thus, the length side of QR is 15 cm. (Djumanta & Susanti, 2008, p. 6)

Table 5. Examples of T_3.

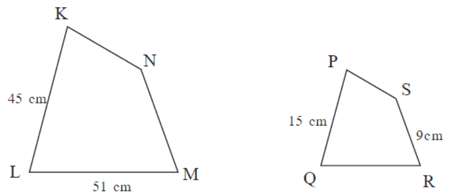
The task in Table 5 asked students to find a missing side in one of two similar rectangles. Thus, it is guaranteed that the two rectangles have proportional corresponding sides. Firstly, students can use technique τ_2 to identify corresponding sides. Secondly, students can compare the lengths of corresponding sides, using the formula $\frac{p_1}{q_1} = \frac{p_2}{q_2}$. Then, students can calculate the missing side by using the cross-product technique from proportion in arithmetic: $p_2 = \frac{p_1}{q_1} . q_2$. The task in Table 5 belongs to the following type of task:
T_3: given two similar polygons P and Q as well as the length p_1 of one side in P and the lengths q_1, q_2 of two sides in Q with p_1 and q_1 being corresponding, find the length p_2 of side in P that corresponds to q_2.
τ_3 : calculate the missing side using $p_2 = \frac{p_1}{q_1} . q_2$.

When two similar polygons are given, the authors also ask students to find a scale factor, as can be seen in Table 6.

T_4: given two similar polygons P and Q as well as lengths of sides $p_1, \ldots, p_n$ in P and lengths of sides $q_1, \ldots, q_n$ in Q. Determine the scale factor between P and Q.

τ_4: k is scale factor if $k = \frac{q_i}{p_i}$ for any corresponding lengths of sides q_i and p_i.

In T_4, Students are given two similar n-gons, where only two sides are known. The authors ask students to find the scale factor. First, students required to identify corresponding sides by using τ_2 (or it can be seen from a figure visually correspond, as in Table 6). In order to get the scale factor, students need to compute $\frac{q}{p}$for the pair (p, q) of corresponding sides. By observing the figure in the Table 6, we agree that the scale factor of KLMN and PQRS is at least 1, so the technique also requires to take p to be the larger of the two sides.



Two similar quadrilaterals KLMN and PQRS are given. Determine the scale factor between KLMN and PQRS.
Answer:
Because KLMN ~ PQRS, the ratio of corresponding sides is equal. It means that$\frac{KL}{PQ} = k$, with k the scale factor. Given KL=45 cm and PQ=15 cm.
So,$\frac{KL}{PQ} = \frac{45cm}{15cm} = 3$. The, the scale factor is $k = 3$.
(Masduki & Budi Utomo, 2007, p. 12)

Table 6. Example of T_4.

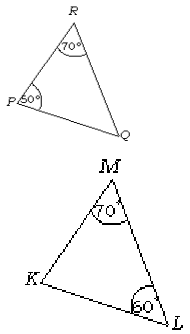
4.3. Practice blocks related to triangles similarity as it commonly appears in Indonesian textbooks

The similarity of triangles involves a number of special techniques, and we found three types of tasks for this case. We now present them as above. In tasks of type T_5, students are given two triangles with given angles and are asked to decide if the triangles are similar; this is solved by verifying the pairwise equality of the given angles (possibly computing one or two angles using that the sum of angles in a triangle is 180^0):

T_5**:** given two triangles S and T as well as their angles $\angle a_1, \angle a_2$ or $\angle a_1, \angle a_2, \angle a_3$ in S and $\angle b_1, \angle b_2$ or $\angle b_1, \angle b_2, \angle b_3$ in T, determine if S and T are similar.

τ_5**:** Under the hypothesis that the angles of each triangle are ordered from the smaller to the larger, check if $\angle a_1 = \angle b_1$, $\angle a_2 = \angle b_2$ and $\angle a_3 = \angle b_3$.

An example can be seen in Table 7.

<table>
<tr>
<td>


</td>
<td>
Consider triangle PQR and triangle KLM from the figure above! Are $\Delta PQR \sim \Delta KLM$?
answer:
$\angle Q = 180^0 - (50^0 + 70^0) = 60^0$
$\angle K = 180^0 - (60^0 + 70^0) = 50^0$
Thus, $\Delta PQR \sim \Delta KLM$
because the corresponding angles are equal
(Wagiyo, Mulyono, Susanto, 2008, p. 13)
</td>
</tr>
</table>

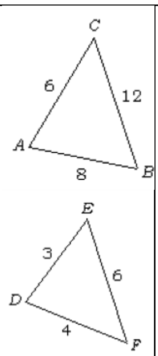
Table 7. Example of T_5

In T_6, students are also asked to decide on similarity. However, in this type of task, students are given the sides of two triangles. Students can order the sides from the shortest one to the longest one. Then, they can check if the corresponding sides of the two triangles have proportional ratio:

T_6: given two triangles S and T as well as sides s_1, s_2, s_3 in S and t_1, t_2, t_3 in T, determine if they are similar.

τ_6: order the sides in the two triangles ($t_1 \leq t_2 \leq t_3$etc.) and check if $\frac{t_1}{s_1} = \frac{t_2}{s_2} = \frac{t_3}{s_3}$.

An example of T_6 can be found in Table 8.

<table>
<tr>
<td>


</td>
<td>
Consider ΔABC and ΔDEFon the picture beside the text!
Are $\Delta ABC \sim \Delta DEF$?
Answer: Proportion of the shortest lengths from two triangles is $\frac{AC}{DE} = \frac{6}{3} = \frac{2}{1}$. Proportion of the longest lengths from two triangles is $\frac{BC}{EF} = \frac{12}{6} = \frac{2}{1}$. Proportion of the third length from two triangles is $\frac{AB}{DF} = \frac{8}{4} = \frac{2}{1}$.Then, $\frac{AC}{DE} = \frac{BC}{EF} = \frac{AB}{DF} = \frac{2}{1}$.
Thus, $\Delta ABC \sim \Delta DEF$.
(Wagiyo, Mulyono & Susanto 2008, p. 12, with voluntarily incorrect shapes of triangles)
</td>
</tr>
</table>

Table 8. Examples of T_6

We also found type of task that asks students to decide on similarity for two triangles. Students are given two corresponding sides and one corresponding angle that is located between these sides. To decide similarity of triangles, students can compare the corresponding angles. Additionally, students need to compare the ratios of the pairs of corresponding sides.

T_7: given two triangles S and T as well as sides s_1, s_2, and $\angle a_1$ that is located between s_1 and s_2 in S and sides t_1, t_2, and $\angle b_1$ that is located between t_1 and t_2 in T. Determine if they are similar.

τ_7: check $\angle a_1 = \angle b_1$ and $\frac{s_1}{t_1} = \frac{s_2}{t_2}$.

An example of T_7 can be seen in the Table 9.

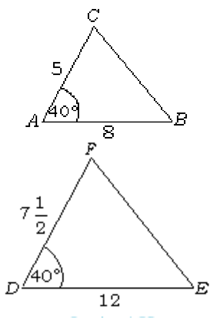
<table>
<tr>
<td>
</td>
<td>Consider figure ΔABC and ΔDEF. Is $\Delta ABC \sim \Delta DEF$?
answer:
$$\angle A = \angle D$$
$$\left.\begin{array}{l}\frac{AC}{DF} = \frac{5}{7\frac{1}{2}} = \frac{2}{3} \\ \frac{AB}{DE} = \frac{8}{12} = \frac{2}{3}\end{array}\right\} \text{two sides that enclose the same angle.}$$
Thus, $\Delta ABC \sim \Delta DEF$ (Wagiyo, Mulyono, Susanto, 2008, p. 13)</td>
</tr>
</table>

Table 9. Example of T_7

Finally, the last task requires τ_3 and some additional algebraic reasoning to identify a missing side in two similar triangles (Table 10).

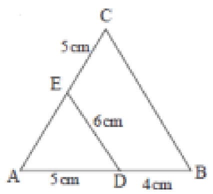
<table>
<tr>
<td>Given Δ ABC~ Δ ADE with DE//BC.
Calculate the length of AE.

</td>
<td>Answer:
we can use proportion of two intersecting lines segments that are intercepted by a pair of parallels DE//BC in Δ ABC:
$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{5}{4} = \frac{AE}{5}$$
$$4 \times AE = 5 \times 5$$
Thus, the length of AE is 6,25 cm
Masduki and Budi Utomo (2007, p. 27)</td>
</tr>
</table>

Table 10. Example of T_8

The technique applied for the question in Table 10 gives rise to

T_8: given the figure 2 as shown with ΔADE~ΔABC, DE//BC, and given the length of three of four sides AE, AC, AD, AB. Find the remaining length.

τ_8: use $\frac{AE}{EC} = \frac{AD}{BD}$ and isolate unknown side.

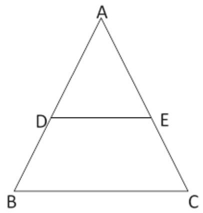


Figure 2. Figure of T_8

4.4. Methodological remarks

In this section, we will discuss practice block of similarity that is rarely occurred in Indonesian textbooks. Thus, it is not significant for quantitative analysis. Normally, this practice block appears as a few tasks from the textbooks which are not easy to categorize into the above types of task. In Table 11, students are given a pair of polygons without specific angles and sides and are asked to decide if they are always similar, maybe similar, or never similar. For example, an equilateral triangle always has sides of the same length. Since equal sides are enough to prove that two triangles are similar, two equilateral triangles are always similar. Additionally, students can also consider that equilateral triangle always has the same angles. This fact is also enough to prove that two triangles are similar Students need to produce 'cases' and use more theoretical knowledge on polygons to solve this task which is difficult to formulate into explicit technique. Thus, we decide to eliminate this task.

Consider the statements below. Write B if the statement is always true, K if the statement is sometimes true and S if the statement is always wrong. Two parallelograms are similar Two equilateral triangles are similar Two rhombuses are similar Two pentagons are similar Sulaiman et al. (2008, p. 7)

Table 11. A task without numbers

In Table 12, students are given only one polygon. Then, they are asked to find another polygon which is similar to the given polygon. We consider leaving this isolated task because this task does not appear in the five others. The task can be formulated as followed and the example can be seen in Table 12.

T: given a polygon P as well as $\angle a_1, \ldots, \angle a_n$ in P and sides p_1, …, p_n in P. Construct another polygon Q which is similar to P.

τ : choose a scale factor k to find the sides $q_1 = k.p_1, \dots, q_n = k.p_n$ if $\angle b_1, \dots, \angle b_n$ in Q are the same angle as $\angle a_1, \dots, \angle a_n$ in P.

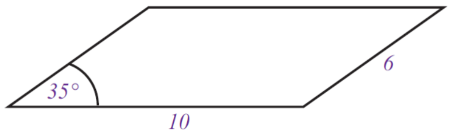
	Construct two other parallelograms that are similar to the given parallelogram. (Agus, 2007, p. 7)

Table 12. A task that is only found in one textbook.

The next example of unclassified task is when students need more than one prior knowledge that is found in only one textbook (Table 13).

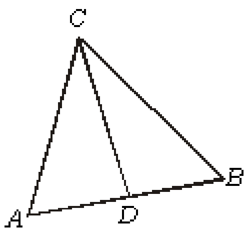
	Consider Δ ABC. CD is a bisector line, prove that $\frac{AC}{BC} = \frac{AD}{DB}$. Wagiyo, Mulyono, Susanto (2008, p. 19)

Table 13. A task that needs more than one prior knowledge.

To solve the task in Table 13, students need to know what a bisector line is, and a visual imaginary to construct two similar triangles. Students also need a parallel line concept (Thales theorem) to prove similarity. In this case, similarity is not only connected to other topic, but also as a way to make a new theory $\frac{AC}{BC} = \frac{AD}{DB}$. However, we leave this task unidentified as it is only found in one textbook. Thus, this task is not very representative of public textbooks.

5. Quantitative survey of types of task in textbooks and in the national examination.

We want to emphasize that Table 14 shows only the numbers of task related to similarity. For each of the six textbooks, we classified all tasks related to similarity into the eight types of tasks (T_1, …, T_8) that we discussed above. In total, we analysed 422 tasks. In six textbooks, some of the tasks do not belong to these eight main types (‘unidentified’ in Table 14). These ‘unidentified’ tasks are typically more theoretical (cf. section 5.3).

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	Unidentified task
Wagiyo et al., grade 9 (2008)	10,4	16,5	49,6	0,9	7,8	4,3	3,5	3,5	5,2
Agus (2007)	11,7	8,8	41,1	0	0	0	17,6	0	20,6
Marsigit et al. (2011)	8,6	10,0	48,6	1,4	15,7	10,0	4,3	0	1,4
Dris, Tasari (2011)	13,8	7,5	65,0	1,25	5,0	1,25	0	0	6,25
Masduki, Budi Utomo (2007)	1,7	8,5	49,0	5,0	13,5	12,0	1,7	1,7	5,0
Djumanta, Susanti (2008)	3,1	14,0	53,1	7,8	6,3	3,1	3,1	0	9,4
	49,3	65,3	306,4	16,35	48,3	30,65	30,2	5,2	47,85

Table 14. A quantitative survey of textbooks analysis (percentage of all tasks).

As we can see in Table 14, there is a considerable variation in how frequently the types of task appear in each textbook. Having such a picture may orient teachers in their choice of textbook. For example, teachers who focus on students' getting a broad knowledge of the theme can choose a textbook that has all types of tasks. Teachers who focus more on advanced tasks can consider textbooks that have more theoretical tasks (unidentified task).

We also notice that in both examples and exercises, there are a few dominant types of task, namely T_3 and T_2. This observation aligns with the national examination tasks in which only T_3 and T_2 appear (Table 15). It is interesting to note that even though the curriculum does not prescribe specific types of task, textbooks seem to align with the national examination to some extent when it comes to the types of tasks they give priority. So, students (and teachers) can rely on the textbook to emphasize preparation for the national examination. At the same time, most of the textbooks cover a broader range of techniques than required at the examination.

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
2008			2					
2009			2					
2010			1					
2011		1	1					
2012			2					
2013		1	1					
2014		1	1					
2015		1	1					

Table 15. A quantitative survey of national examination.

6. Connections in the textbooks between similarity and proportion.

The analysis and comparison of textbooks obviously cannot limit itself to the analysis of practices in independent sectors and domains. Connections between domains are crucial for the students to learn mathematics as a coherent discipline. If we compare the practice blocks of proportion in arithmetic (Wijayanti & Winsløw, 2015) with the practice blocks of similarity in geometry (section 5.2), we notice that proportion and similarity share similar techniques:

T_3: Given two similar n-gons P and Q as well as one side p_1 in P and two sides q_1, q_2 in Q with p_1 and q_1 being corresponding, find the side p_2 in P that corresponds to q_2.
τ_3: calculate the missing side using $p_2 = \frac{p_1}{q_1} . q_2$.

This "missing side (T_3)" practice is very similar to a practice block which Wijayanti & Winslow (2017) found to be dominant in Indonesian textbooks' treatment of ratio and proportion in arithmetic (7th grade), namely the following:

T: given a pair of numbers (x_1, x_2) and a third number y_1 find y_2 so that $(x_1, x_2) \sim (y_1, y_2)$
τ : calculate $y_2 = \frac{x_2 . y_1}{x_1}$.

The two types of tasks have almost the same technique. The difference is that in the tasks of the similarity sector, figures are used to present the numbers coming from dimensions in figures, while in the proportion tasks; students are given numbers using quantities like price, weight, length and so on. In both cases, students often have to infer from the situation - geometrical or quantitative - that similarity, respectively proportionality, can be assumed.

The treatment of arithmetic in 7th grade also includes a type of task which is very similar to T_4 above, namely the following (Wijayanti & Winsløw, 2015):

T: given x_1 and x_2, find r so that $(x_1, x_2) \sim (1, r)$.
τ : $r = x_2 / x_1$ (finding the ratio).

The closeness of techniques from proportion and similarity suggest that it is didactically useful to build connections between them. However, we suppose that these potential connections might not be established for students, unless textbooks point them out explicitly, either by the text declaring a 'closeness' of the two sectors in general, by the use of specialized terms like 'proportion' in geometry, or directly at the time of explaining techniques such as the above.

As regards connection at the level of sectors, we focus on statements in the chapter on similarity that refer explicitly to the sector of proportion in 7th grade arithmetic.

There is such an explicit statement in three textbooks (Djumanta & Susanti, 2008; Dris & Tasari, 2011; Marsigit et al., 2011). One example is in Figure 3:

Masih ingatkah kalian tentang konsep perbandingan yang telah dipelajari di kelas VII? Konsep perbandingan ini harus kalian pahami terlebih dahulu sebelum mempelajari konsep kesebangunan bangun datar, karena pada pembahasan konsep kesebangunan akan berhubungan dengan perbandingan.

Figure 3. One example of explicit statement (Dris and Tasari, 2011, p. 1)

'Do you still remember about the proportion concept that you have learnt in the 7th grade? The proportion concept is needed before we learn about polygon similarity, because similarity relates to proportion' (translated in English).

Declarations such as above merely indicate the existence of some connections between sectors. However, it says nothing about what this connection means. To clarify this situation, we will also elaborate more on the sector level. Proportion as a sector can be divided into two themes, namely ratio (scale) and proportion (Wijayanti & Winsløw, 2015). The appearance of these themes in similarity would mean that part of the technology of these themes is invested in the textbooks' discussion (technology) of similarity techniques. Concretely, we have looked for the use of specialized terms like 'scale' and 'proportion'. We found that all of the textbooks use the term of 'proportion' in defining similarity (an example in Figure 4).

Dua segitiga dikatakan sebangun jika sisi-sisi yang bersesuaian sebanding atau sudut-sudut yang bersesuaian sama besar.

Figure 4. One example of definition of similarity (Djumanta and Susanti, 2008)

'Two triangles are similar if the corresponding sides are proportional or the corresponding angles are equal' (translated in English).

Secondly, we found the term 'scale' in five textbooks (Djumanta & Susanti, 2008; Dris & Tasari, 2011; Marsigit et al., 2011; Masduki & Budi Utomo, 2007; Wagiyo et al., 2008). In the example, scale is used to relate proportion and similarity:

> '*Different sizes of a picture* can be obtained by enlarging or reducing the original picture by a certain factor. Thus, *pictures of different size* have the same shape and proportional sides. The numbers of proportional sides is often found using a scale. In other words, *the enlarged (or reduced) picture* and the original picture are similar'. (Translated in English, Wagiyo et al., 2008, p. 1)

Of course, the most concrete and direct connection between the sectors occur at the level of practice, that is when techniques from proportion are used for solving tasks on similarity. However, no textbooks explicitly point out such a connection e.g. in examples of how to solve task on similarity. But two textbooks (Djumanta & Susanti, 2008; Dris & Tasari, 2011) suggest such connection by investing a section called 'prerequisite competences' before treating examples of similarity task; this extra section simply shows some examples of proportion task of the types treated in grade 7. This way the authors suggest (without further explanation) that certain 'familiar' tasks and techniques are useful to recall before approaching the new types of task.

7. Conclusion

This study shows one way in which the anthropological theory of the didactic can be applied to analyse textbooks. As the theory part in the lower secondary textbooks is limited, we can mainly observe how the notion of similarity is introduced and then worked within tasks. We found that the authors focus more on how to use similarity to solve tasks than on treating in the notion itself.

The chapter on similarity always start with an informal definition of what it means for two (general) polygons to be similar. This reflects, in fact, the order suggested by the curriculum. We constructed a reference model based on the most common task on similarity found in the textbooks. We found a few tasks that cannot be categorized using these eight types of tasks but these are only found in a few textbooks. Thus, our reference model captures essentially the practical blocks found in the analysed texts. The eight types of task fall in two parts: polygon similarity and triangle similarity. We also found that the same types of tasks dominate in the textbooks and appear as dominative types of tasks in the national examination. Finally, we conclude that on the reference model can help us to see a potential connection between proportion and similarity. A few books explicitly establish connection at the level of the sectors and at the level of subjects (individual technique), while all have at least some explicit connection at the level of themes (use of terms). As we observed in the introduction section, connections in the textbooks maybe important to help students to see mathematics as connected body of knowledge.

Above all, this study wants to show how the ATD framework can be used to do comparative textbooks analysis. In fact, this framework allows us to compare the student's activity proposed by textbooks in a somewhat neutral way based on epistemological analysis. In other words, the epistemological analysis is insufficient for an educational purpose. As a result, teachers can also apply the results of this study to choose what textbook to use for students to relate to the praxeologies of similarity.

After analysing the organization of the practice blocks, the analysis of themes becomes possible. Again, the ATD approach helps the researchers to see the connection or disconnections in a more objective way. Finally, the reference epistemological model was used to analyse the textbooks in relating to official documents such as the national examination and curriculum. This shows the role the national examination plays supplementary guideline for textbook authors and teachers when it comes to the types of task that they give priority.

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