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UNPACKING THE CALCULUS TRANSITION: A MULTI-DIMENSIONAL VIEW

Abstract. This introductory paper presents the aims and structure of the special issue. The seven contributions gathered in the issue explore secondary-tertiary calculus transition through three thematic clusters: cognitive development and conceptual foundations, epistemological and instructional continuity, and curriculum structure and pedagogical proposals. This paper situates these studies within current international research trends, emphasizing the need for epistemological unity, representational fluency, and teacher empowerment to foster coherent and meaningful calculus learning trajectories.

Key words. Calculus transition, conceptual understanding, epistemological continuity, curriculum design, teacher knowledge.

Résumé. Décrypter la transition en analyse réelle : une perspective multidimensionnelle. Cet article introductif présente les objectifs et la structuration du numéro spécial. Les sept contributions réunies dans ce numéro sont organisées selon trois axes thématiques : le développement cognitif et les fondements conceptuels, la continuité épistémologique et didactique, et la structuration curriculaire et les propositions pédagogiques. L'article inscrit ces contributions dans le paysage actuel de la recherche en didactique des mathématiques, en soulignant l'importance de l'unité épistémologique, de la fluidité représentationnelle, ainsi que du renforcement du pouvoir d'agir des enseignants pour soutenir des trajectoires d'apprentissage de l'analyse réelle cohérentes et porteuses de sens.

Mots-clés. Transition en analyse réelle, développement cognitif, continuité épistémologique, conception curriculaire, connaissances des enseignants.

The transition between secondary and tertiary education in calculus is widely recognized as a critical and often challenging phase in students' mathematical trajectories. Foundational concepts such as limits, derivatives, and integrals become not only more formalized but also demand new forms of reasoning and epistemological engagement. This special issue assembles a collection of seven articles that, together, provide a comprehensive and nuanced exploration of this transition. Each contribution sheds light on different dimensions of this complex phenomenon—from cognitive development and epistemological continuity to curriculum structure and pedagogical proposal.

To better interpret and engage with the findings, the contributions were grouped into three thematic clusters: (1) cognitive development and conceptual foundations, (2) epistemological and instructional continuity, and (3) curriculum structure and

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pedagogical proposal. These clusters reflect both established and emerging trends in mathematics education research related to the secondary-tertiary calculus transition and offer implications for curriculum design, teacher education, and instructional practice.

1. Outline of the contributions

1.1. Cognitive development and conceptual foundations

Two contributions—by Falach et al., and Thomas et al.—focus on how students build, consolidate, and shift between intuitive and formal understandings of central calculus concepts. These studies highlight the developmental nature of conceptual knowledge and suggest that instructional design should better scaffold transitions between informal reasoning and formal definitions.

Falach et al. "Accumulative thinking as an Intuitive base for the concept of integral" examines the notion of accumulative thinking as a cognitive base for integration. Drawing on the "Abstraction in Context" framework (Dreyfus et al., 2015), their study shows how Grade 11 students, through tasks involving real-life contexts such as water flow into a pool, can construct meaningful precursors to the concept of the definite integral. Their findings support the view—also advanced by Thompson (2022) and Ely & Jones (2023)—that reasoning grounded in rate-of-change and accumulation provides a cognitively accessible pathway into integral calculus. This aligns with recent calls to design curricula that introduce integration from an intuitive, visual and dynamic standpoint before shifting to formal symbolic representations.

Thomas et al. "Building thinking about graphical antiderivatives: The role of interval perspectives" explores the development of students' thinking about graphical antiderivatives. Their study emphasizes the role of interval-based perspectives, which allow students to see antiderivatives as families of functions determined by constant differences. This approach aligns with research highlighting the importance of graphical and geometric reasoning (Tall, 1992) and supports the argument that visual intuition can function as a conceptual bridge from secondary-level tasks to tertiary-level abstraction.

1.2. Epistemological and instructional continuity

The second thematic strand, containing three contributions, emphasize the importance of foundational epistemological understanding, advocating for more instructional continuity in the secondary-tertiary transition. Harel, Durand-Guerrier et al., and Bridoux examine how epistemological and instructional choices may mediate or hinder students' conceptual progression.

Harel "Calculus education: aspects of order, continuity, and reconceptualization" discusses three dominant instructional models in calculus education—DI (Differentiation first), ID (Integration first), and TI (Thompson's integrated approach, see Project DIRACC in http://patthompson.net/ThompsonCalc/)—and evaluates their implications for the continuity of learning. He underscores the disruptive effects of curricular incoherence across educational levels, echoing previous findings that inconsistencies in concept sequencing can produce significant cognitive dissonance in students. His analysis resonates with recent calls for reconceptualizing calculus instruction to prioritize coherence and epistemic alignment.

Bridoux "Specificity of limit notion in Belgian secondary school: Potentialities to foster calculus teaching in the university" offers a detailed analysis of how the concept of limit is introduced in the Belgian secondary school curriculum, paying particular attention to the epistemological nature of limit as a formalizing and generalizing construct (Robert, 1998). Her study illustrates how carefully designed curricular elements—such as the use of quantifiers and multiple semiotic registers (Duval, 1993)—can support students in navigating the conceptual shift required by university-level calculus. Notably, Bridoux introduces the notion of discursive proximities to show how textbook design either facilitates or hinders students' opportunities to make epistemological connections.

Durand-Guerrier et al. "On the role of order in Calculus at the secondary-tertiary transition" turns to foundational epistemological issues, focusing on the role of order in the real number system and its implications for continuity, connectedness, and completeness in calculus. They argue for introducing students to the structural aspects of the number system earlier, thus allowing for a more meaningful engagement with formal calculus concepts. This builds on a line of inquiry emphasizing the centrality of logical and structural reasoning in advanced mathematical thinking.

1.3. Curriculum structure and pedagogical proposal

The final group of articles, consisting of two contributions, addresses different but complementary aspects of improving the secondary–tertiary transition in calculus education. Heggelund et al. bring into focus the structural and curricular disjunctures that characterize this transition in the case of the integral. Sosa and Almaraz focus on deepening the conceptual understanding of high school teachers regarding the Fundamental Theorem of Calculus, proposing theoretical models aligned with university expectations. Together, these works offer complementary insights into the challenges of the integral in this transitional phase, from curriculum alignment to teacher conceptualization.

Heggelund et al. "The didactical transposition and praxeology related to the study of integration at the secondary and tertiary levels in Norway" investigates the case of Norway, where integration is taught using an intuitive infinitesimal approach in secondary school but is formalized via Riemann sums and limit-based reasoning at university. Applying the theoretical lens of didactical transposition (Chevallard, 1992), the study reveals how the epistemological distance between the two educational levels leads to fragmented student conceptions. It also highlights how institutional representations (textbooks, syllabi) influence the praxeologies students acquire, often leaving gaps in foundational understanding. These findings underline the importance of alignment not just in content, but in the epistemological underpinnings and representations of concepts.

Sosa and Almaraz "Design and implementation of a teaching proposal of the fundamental theorem of calculus to explore the conjunction of Mathematics Teacher´s Specialized Knowledge (Carrillo et al., 2017) and Teaching for Robust Understanding" exemplifies a pedagogical proposal by designing and implementing a high school teaching proposal for the Fundamental Theorem of Calculus grounded in the mathematics teacher's specialized knowledge model and the teaching for robust understanding framework (Schoenfeld et al., 2016). Using an action-research methodology, the authors explore how these theoretical constructs can be jointly mobilized to transform traditional instruction and promote deeper conceptual understanding, thus aligning secondary education with the expectations of tertiary mathematics learning. The study highlights how secondary instruction can be reimagined to reflect the depth and complexity typically expected at the university level, positioning this study within a broader effort to rethink teacher knowledge and classroom practice in transitional educational stages.

2. Situating the contributions in contemporary research

The seven contributions in this special issue are emblematic of current developments in research on the secondary-tertiary transition in mathematics education, particularly in calculus where there is a growing focus on understanding how conceptual knowledge develops, how curricula are structured, and how teachers can innovate in response to the needs of their students. This research domain has evolved significantly over the past two decades, shifting from a narrow focus on cognitive difficulties toward a more integrated perspective that encompasses cognitive, epistemological, curricular, and instructional aspects (Di Martino, Gregorio, & Iannone, 2023; Gueudet et al., 2016). The seven contributions emphasize the importance of continuity between secondary and tertiary education in calculus, particularly in the way foundational concepts like limits, continuity, and integration are taught. These articles provide valuable insights into how this continuity can be achieved through curricular redesign, teachers' knowledge, and a more cohesive approach to pedagogy. In what follows, we highlight several structuring dimensions that situate this special issue within the contemporary research landscape:

- From cognitive barriers to epistemological access: Traditional studies emphasized students' cognitive obstacles in grasping formal definitions, particularly around limits and continuity. This issue contributes to a growing body of work that reframes these challenges as stemming not merely from cognitive perspective but also from epistemological aspects in how concepts are framed and taught at different levels (Falach et al., Thomas et al.; Harel; Bridoux; Durand-Guerrier et al.). The Brousseau's idea of the fundamental situation (Brousseau, 1998) becomes particularly relevant here, emphasizing the need for learning opportunities rooted in meaningful access to the mathematical ideas. This access is mainly funded on an epistemological analyze in terms of invariant of knowledge (Brousseau, 1997) and *raison d'être* of knowledge (Rogalski, 1986).

- Curricular coherence and institutional structures: Papers such as those by Bridoux; Harel; Heggelund et al. address the role of curricular and institutional misalignments—in sequencing, language, representations, or expectations— as key sources of disruption in students' learning trajectories. This resonates with prior research emphasizing the need to address such forms of discontinuity to promote curricular uniformity between school and university mathematics (Biza et al., 2016; Hochmuth et al., 2021).

- Teacher knowledge perspective: This special issue also contributes to the growing recognition of teacher knowledge as levers for change (e.g., Sosa and Almaraz). Similarly, to other efforts, this special issue highlights how teacher knowledge, far from being a mere vehicle for content delivery, can drive curricular change and support conceptual continuity between secondary and university-level calculus. In doing so, it reinforces the idea that empowering teachers with robust theoretical and pedagogical tools is key to bridging the secondary–tertiary divide in meaningful and sustainable ways (e.g., Kajander et Colgan, 2024; Trouche et al., 2020).

- Semiotic and representational transitions: the role of semiotic registers (Duval, 1993) in supporting students' conceptual development is well known: moving fluently between registers is a main challenge in the learning. It is a key challenge in the calculus learning since many registers are often involved in the same time: graphical, formal, natural language, and algebraic registers. Papers such as those by Bridoux; and Thomas et al. explain how this semiotic study can be useful when deployed with other framework.

3. Looking ahead: research and practice directions

The contributions gathered in this issue provide productive ground for reconceptualizing the secondary-tertiary transition as a multi-layered, longitudinal, and institutionally embedded process. Moving forward, several strategic directions for research and practice can be proposed:

- Designing for epistemological continuity: Future work should focus on designing didactical situations and sequences that preserve conceptual coherence across educational levels. Didactical engineering (Artigue, 1995) and theory-informed task design (Brousseau, 1997) offer productive methodologies in this regard. Curricula should aim to explicitly build bridges between intuitive, procedural, and formal understandings, especially for FUG-type concepts (Robert, 1998), such as limits and integrals.

- Empowering teachers through professional development: Given the pivotal role teachers play in mediating the curriculum, supporting professional development that fosters deep epistemological understanding and reflective practice is essential. Boundary-crossing initiatives (Akkerman & Bakker, 2011), communities of practice (Wenger, 1998), and joint curriculum development (Clark-Wilson et al., 2015) offer promising frameworks.

- Expanding the role of representational fluency: There is a need for empirical studies investigating how students transition between representational registers—particularly as they encounter advanced definitions and proofs. Further work should explore how representational fluency mediates access to formal calculus and real analysis.

- Addressing the affective and identity dimensions: In line with emerging trends (Di Martino & Zan, 2011), future research should more systematically examine students' emotional, motivational, and identity-related experiences during the transition. Understanding how students position themselves as learners of mathematics—and how institutional practices shape this positioning—can illuminate important barriers and affordances.

- Toward longitudinal and comparative studies: Finally, the field would benefit from more longitudinal research designs that trace students' evolving understanding over time and comparative studies that explore how national contexts and institutional cultures shape the transition. Such approaches can enrich our understanding of which practices are locally adapted and which are more universally applicable.

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