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Teaching and learning of calculus: duality of the transition between secondary and tertiary education.

IREM de STRASBOURG

ABSTRACTS

IMÈNE GHEDAMSI, THOMAS LECORRE, OLOV VIIRMAN. Unpacking the calculus transition: A multi-dimensional view

This introductory paper presents the aims and structure of the special issue. The seven contributions gathered in the issue explore secondary-tertiary calculus transition through three thematic clusters: cognitive development and conceptual foundations, epistemological and instructional continuity, and curriculum structure and pedagogical proposals. This paper situates these studies within current international research trends, emphasizing the need for epistemological unity, representational fluency, and teacher empowerment to foster coherent and meaningful calculus learning trajectories..

GILAT FALACH, ANATOLI KOUROPATOV, TOMMY DREYFUS. Accumulative Thinking as an intuitive base for the concept of integral

Integral calculus presents persistent challenges for students in general and for those transitioning from secondary to tertiary education in particular. This study examines the development of "Accumulative Thinking" as a foundation for understanding integration. In a purposely designed learning activity, pairs of Grade 11 students explored accumulation concepts using the context of water flowing into a pool. Using the Abstraction in Context framework, we analysed students' processes of constructing knowledge during this activity. Our findings indicate that most students constructed elements of Accumulative Thinking, preparing them for future studies; the findings also demonstrate how real-world contexts can facilitate the development of Accumulative Thinking.

MIKE THOMAS, TOMMY DREYFUS, CAROLINE YOON. Building Thinking about Graphical Antiderivatives: The Role of Interval Perspectives

The concept of function traverses the mathematics studied at school and university and is an important transitional link between the two. We examine how an interval perspective on function may help with constructing graphical antiderivative functions. In doing so a number of important constructs needed to build such a graphical understanding are considered, along with how the students in the study linked them and built with them. In addition, some of the difficulties they faced and possible reasons for them are explained. The evidence presented shows that an interval perspective on function was important in being able to construct antiderivative functions graphically. Hence, we propose that this interval perspective on function may prove useful in helping students in transition to construct a local perspective on function. In turn we suggest what a potential path for thinking about graphical antiderivatives could look like and the kind of activities that could assist transition students along it.

GUERSHON HAREL. Calculus Education: Aspects of Order, Continuity, and Reconceptualization

The triad order, continuity, and reconceptualization that appears in the title of this paper refers to a juxtaposition of three aspects of calculus education. Order refers to differentiation followed by integration (DI approach) versus integration followed by differentiation (ID approach) versus Thompson's integrated approach (TI approach) which view differentiation and integration inseparable. Continuity refers to the impact of these approaches on student learning as they transition from high school to university. Reconceptualization refers to the effort to reform calculus learning and teaching by reeducating future secondary teachers relearn calculus concepts and ideas through the lens of quantitative reasoning. This is an analytic paper. It begins with an analysis of the cognitive and pedagogical features of the three approaches, DI, ID, and TI, and continues with a discussion of the continuity problem concerning the transition from school mathematics to university mathematics, focusing on the difficulty to reform calculus education in the U.S. To advance this reform, it is necessary to examine in depth the current approaches to calculus education in the U.S., as well as alternative approaches advocated by mathematicians and mathematics education scholars. The analysis of the three approaches, DI, ID, and TI, aims at contributing to this essential examination. As part of this examination, the paper offers a calculus module for prospective secondary teachers who have already taken the "mainstream" calculus sequence. The module, while akin to the TI approach, its development and implementation rest on a separate theoretical framework.

STÉPHANIE BRIDOUX. Spécificités de la notion de limite dans l'enseignement secondaire belge : quelles potentialités pour favoriser l'enseignement de l'Analyse à l'université ?

Specificity of limit notion in Belgium secondary school: which potentialities to foster Calculus teaching at university?

This study concerns the teaching of Calculus in secondary school, with a focus on the introduction of the limit notion (sequences and functions). Prior research has highlighted recurrent difficulties of first-year university students with this notion. We try to show that the secondary school curriculum in French-speaking Belgium offers many continuities between high school and university. However, the analysis of a textbook shows discrepancies with the potential identified in the curriculum.

VIVIANE DURAND-GUERRIER, GAËTAN PLANCHON, NICOLAS SABY. Sur le rôle de l'ordre en analyse à la transition secondaire – supérieur.

On the role of order in Calculus at the secondary-tertiary transition

In France, current curricular choices tend to make the role of order in analysis invisible. They implicitly favour a topological point of view related to metric spaces, which we find in the notion of absolute value, generalised to that of distance. These choices lead us to leave aside a complementary perspective that emphasises ordered sets, in which the connected parts of \mathbb{R} , which are intervals, play a central role. In our epistemological study and using two examples from the secondary-tertiary transition, we show that considering the relationship between connectedness and completeness in the set of real numbers encourages an explicit consideration of the role of order in analysis, especially in proofs. Finally, we suggest some avenues of research opened up by this work to help take into account the dual nature of this transition.

KRISTOFFER HEGGELUND OMARHAUG, HANS KRISTIAN NILSEN. The Didactical Transposition and Praxeology related to the study of Integration at the Secondary and Tertiary levels in Norway

This paper builds on two studies conducted in Norway, respectively with students in their last year at upper secondary school and engineering students in their first year at university. From the perspective of the Anthropological Theory of the Didactic (ATD), we discuss the didactical transposition concerning the topic of integration relevant for both institutions. The curricula do not favour any one specific approach to integration, and Riemann-sums dominate the textbook used in the university, while an infinitesimal-based approach was offered at upper secondary. The praxeology at upper secondary school seems to lead students exclusively towards anti-derivative and area-interpretations of the integral, while the university students show more varied infinitesimal-based interpretations. We argue a need for revising how integrals are presented and worked with, at both levels of schooling.

LETICIA SOSA, CINTHIA ALMARAZ. Design and Implementation of a teaching proposal of the Fundamental Theorem of Calculus to explore the conjunction of Mathematics Teacher's Specialized Knowledge and Teaching for Robust Understanding

The aim of this research is to design and implement a teaching proposal of the Fundamental Theorem of Calculus at the high school level, to explore the conjunction of the categories of Mathematics Teacher's Specialized Knowledge (MTSK) model and the dimensions of Teaching for Robust Understanding (TRU). MTSK model and TRU have been used as a framework of reference. The research is a qualitative, exploratory study within an interpretive paradigm, employing the action research method. Subsequently, the data analysis is described through five stages. Finally the results give an approach of the conjunction of MTSK and TRU, we identified elements from the subdomains of the MTSK that can be related to some dimensions of the TRU. However, there are other elements from each model for which relation has not been clear, both in theory and in practice.